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One-loop renormalization of Resonance Chiral Theory: scalar and pseudoscalar resonances*

Ignasi Rosell,[a] **Pedro Ruiz-Femenía**[b] **and Jorge Portolés**[a]

[a] *Departament de Física Teòrica, IFIC, CSIC — Universitat de València*
Edifici d'Instituts de Paterna, Apt. Correus 22085, E-46071 València, Spain
[b] *Theory Division, Max-Planck-Institut für Physik*
Föhringer Ring 6, D-80805 München, Germany
E-mail: Ignasi.Rosell@ific.uv.es, ruizfeme@mppmu.mpg.de, jorge.portoles@ific.uv.es

Abstract: We consider the Resonance Chiral Theory with one multiplet of scalar and pseudoscalar resonances, up to bilinear couplings in the resonance fields, and evaluate its β-function at one-loop with the use of the background field method. Thus we also provide the full set of operators that renormalize the theory at one loop and render it finite.

Keywords: 1/N Expansion, Chiral Lagrangians, Nonperturbative Effects, QCD.

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Contents

1. Introduction

It is well known that the study of low-energy hadrodynamics is tampered with by our present inability to implement non-perturbative Quantum Chromodynamics (QCD) fully in those processes. Hence, conspicuously, in order to disentangle New Physics effects from the Standard Model we need to work out a frame that enforces QCD in this energy region as close as possible.

In the very low-energy regime (typically $E \ll M_V$, where M_V is short for the mass of the lightest vector meson multiplet) Chiral Perturbation Theory (χPT) [1, 2] has become a successful model-independent tool that exploits the main features of an Effective Theory approach to QCD, namely the symmetries of the underlying theory and the phenomenologically suited assumption of spontaneous breaking of the chiral symmetry. χPT is a non-decoupling Effective Theory ruled by the $\mathrm{SU}(N_F)_L \otimes \mathrm{SU}(N_F)_R$ chiral symmetry of massless QCD and it is built in terms of the pseudo-Goldstone bosons (to be identified with the lightest multiplet of pseudoscalar mesons) that are generated in the phase transition driven by the spontaneous breaking of the symmetry, all the heavier states being integrated out. This framework is worked out as a perturbative expansion in the momenta and masses of the pseudoscalar mesons and it has proven to be a rigorous and fruitful scheme [3]. Although χPT is non-renormalizable *stricto sensu*, it supports a perturbative renormalization procedure where all loop divergences can be reabsorbed (when regularized within a scheme that respects chiral symmetry) into a finite number of new operators. This procedure is well defined order by order in the chiral expansion. Accordingly its generating functional has been systematically calculated up to $\mathcal{O}(p^6)$, that is two loops for even-intrinsic-parity processes [2, 4, 5] and one loop in the odd-intrinsic-parity sector [6]. χPT has profusely guided light meson dynamics in the last twenty years.

At higher energies ($M_V \lesssim E \lesssim 2\,\mathrm{GeV}$) the situation is more involved. This regime is populated by many resonances and the absence of a mass gap in the spectrum of states makes difficult to provide a formal Effective Theory approach to implement QCD properly. Hence we have to rely on additional information provided by the strong interaction underlying theory. Large-N_C QCD [7] furnishes a practical scenario to work with. The $N_C \to \infty$ limit strongly constrains meson dynamics by asserting that the Green Functions of the theory are described by the tree diagrams of an effective local lagrangian with local vertices and meson fields, higher corrections in $1/N_C$ being yielded by loops described within the same lagrangian theory. In addition the spectrum of this limit of QCD is given by an infinite set of non-decaying meson states. This scheme has supplied thorough insights to extract information from the theory [8, 9]. Nevertheless a strict formulation of large-N_C QCD in the $N_C \to \infty$ limit is still lacking, mainly due to the fact that we do not know how to implement an infinite spectrum in a model-independent manner.

The known phenomenology of hadron processes seems to support the assumption that when a resonance leads the dynamics of an observable, heavier resonances, with the same quantum numbers, tend to play a decreasing role. This reasonable endorsed conjecture helps us to model large-N_C QCD with a lagrangian theory that involves, besides pseudoscalar mesons, one U(3) multiplet of scalar, pseudoscalar, vector and axial-vector resonances, each one to be identified with the lightest corresponding states in the phenomenological spectrum. The remaining restrictions on the theory come from the symmetry properties of QCD, like chiral symmetry driving the interactions of the pseudoscalar mesons and U(3) unitary symmetry for the matter fields (resonances) [10]. This construction, that constitutes the basis of the Resonance Chiral Theory (RχT), was carried out in detail in ref. [11], though considering operators with only one resonance field in the even-intrinsic-parity sector.[1] Notice that in contrast to many models of the resonance fields that have been widely employed in the literature, RχT only uses basic QCD symmetry features without any additional *ad hoc* assumptions. Its model aspect only comes from the fact that we do not include an infinite spectrum in the theory, which is one of the features of the $N_C \to \infty$ limit of QCD. Finally the elements that provide the theory, symmetries and spectrum, give the structure of the operators but do not supply information on the couplings of the lagrangian. However these are strongly constrained by short-distance properties of the un-

[1]An extension to operators contributing, upon integration, to the $\mathcal{O}(p^6)$ χPT lagrangian is under way [12].

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derlying QCD dynamics: an interplay between hadronic form factors and Green functions of the QCD currents can furnish all-important information on the couplings [13] through a matching procedure that has been very much used lately with notable success [14, 15]. Other models have also been used to fulfill this task [16].

Since its inception RχT has been applied both to the study of resonance contributions in weak interaction processes (radiative and non-leptonic kaon decays) [17] and to the study of form factors of mesons [18] where only the RχT lagrangian at tree level has been used and, accordingly, the leading contribution in the large-N_C model we are describing has been obtained. The next-to-leading order in the $1/N_C$ expansion arises from one loop calculations within the theory and its control starts to be necessary both on grounds of the convergence of the predictions and to straighten our knowledge of non-perturbative QCD. Some pioneering work at one-loop has already been performed [19, 20] showing not only the technical difficulties that appear but also the conceptual intricacies that involve the construction of the theory.

RχT is non-renormalizable. Moreover the lack of an expansion parameter in the lagrangian does not make feasible the application of a perturbative renormalization program based on a well defined power-counting scheme analogous to the one in χPT. Nevertheless from a practical point of view the situation is similar to the χPT case [21]. As shown in ref. [20], where the vector form factor of the pion was calculated at one loop in RχT, it is possible to construct a finite number of operators, within the theory, whose couplings can absorb the divergences coming from one loop diagrams. The only requirement is, of course, that the regularization procedure of the loop divergences respects the symmetries of the lagrangian.

In the present article we have studied the full one-loop generating functional that arises from RχT when one multiplet of scalar and pseudoscalar resonances are considered and only up to bilinear couplings in the resonances are included. We have evaluated the divergent contributions and, consequently, we have obtained the full set of operators needed to renormalize the theory properly. The conceptual differences with the χPT renormalization program will also be stressed.

In section 2 we describe shortly the content of RχT that is of interest in our case. Section 3 is devoted to explain the procedure and hints that we follow to perform the evaluation of the generating functional, whose results are given in section 4 and commented in section 5. In section A we point out the conclusions and summarize. Most of the technical details are relegated to the appendices.

2. RχT with scalar and pseudoscalar resonances

We consider the RχT lagrangian constituted by pseudo-Goldstone bosons (the lightest pseudoscalar mesons) and one multiplet of both scalar and pseudoscalar resonances. Motivated by the large-N_C limit we include U(3) multiplets for the spectrum, i.e. both for the pseudo-Goldstone bosons and the resonances, though we limit ourselves to SU(3) external currents as we are not interested in anomaly related issues. Our lagrangian reads:

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}(\phi, \mathrm{S}, \mathrm{P}) = \mathcal{L}_\chi^{(2)} + \mathcal{L}_{\mathrm{kin}}(\mathrm{S}, \mathrm{P}) + \mathcal{L}_2(\mathrm{S}) + \mathcal{L}_2(\mathrm{P}) + \mathcal{L}_2(\mathrm{S}, \mathrm{P}) , \tag{2.1}$$

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where $\mathcal{L}_\chi^{(2)}$ is the $\mathcal{O}(p^2)$ χPT lagrangian,

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \, , \tag{2.2}$$

with $F \simeq 92.4\,\mathrm{MeV}$ the decay constant of the pion in the chiral limit and the brackets $\langle \cdots \rangle$ stand for a trace in the flavour space. The kinetic term for the scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances is:

$$\mathcal{L}_{\mathrm{kin}}(\mathrm{S,P}) = \frac{1}{2} \sum_{R=S,P} \left\langle \nabla^\mu R \, \nabla_\mu R - M_R^2 \, R^2 \right\rangle , \tag{2.3}$$

that also includes interaction terms through the covariant derivatives. Finally the pure interacting terms for resonances and pseudoscalars are given by [11, 12]:

$$\begin{aligned}
\mathcal{L}_2(\mathrm{S}) &= c_d \, \langle S \, u_\mu u^\mu \rangle + c_m \, \langle S \, \chi_+ \rangle + \lambda_1^{\mathrm{SS}} \, \langle SS \, u^\mu u_\mu \rangle + \lambda_2^{\mathrm{SS}} \, \langle S u_\mu S u^\mu \rangle + \lambda_3^{\mathrm{SS}} \, \langle SS \, \chi_+ \rangle \, , \\
\mathcal{L}_2(\mathrm{P}) &= i \, d_m \, \langle P \, \chi_- \rangle + \lambda_1^{\mathrm{PP}} \, \langle PP \, u^\mu u_\mu \rangle + \lambda_2^{\mathrm{PP}} \, \langle P u_\mu P u^\mu \rangle + \lambda_3^{\mathrm{PP}} \, \langle PP \, \chi_+ \rangle \, , \\
\mathcal{L}_2(\mathrm{S,P}) &= \lambda_1^{\mathrm{SP}} \, \langle \{ \nabla_\mu S, P \} u^\mu \rangle + i \, \lambda_2^{\mathrm{SP}} \, \langle \{ S, P \} \chi_- \rangle \, ,
\end{aligned} \tag{2.4}$$

where all the couplings are real valued. We follow closely the notation of refs. [11, 12] that, for convenience, is recalled in appendix A.

Several comments on our lagrangian theory[2] are suitable here:

- Notice that we are not including the χPT lagrangian of $\mathcal{O}(p^4)$ and higher orders. It has been shown [11] that $\mathcal{L}_\chi^{(4)}$ is largely saturated[3] by the resonance exchange generated by the linear terms in the resonance field given by $\mathcal{L}_2(S)$ and $\mathcal{L}_2(P)$, hence the explicit introduction of $\mathcal{L}_\chi^{(4)}$ would amount to include an overlap between both resonance contributions. An analogous analysis at $\mathcal{O}(p^6)$ has not been performed systematically but it also looks a reasonable assumption. Thus our theory stands for a complete resonance saturation of the χPT lagrangian.

- The structure of the resonance interacting operators follows a definite pattern. The linear terms in the resonance fields were already introduced in ref. [11] and have the structure $\langle R \, \chi^{(2)} \rangle$, where $\chi^{(2)} \equiv \chi_\pm$, $u_\mu u^\mu$ is a chiral tensor, involving Goldstone bosons and external currents, of $\mathcal{O}(p^2)$ in the chiral counting.[4] However the theory does not look consistent with these pieces only. This is because the kinetic term in eq. (2.3), due to the covariant derivative, also includes an interacting term with two resonances. Hence it seems congruous to include all the interacting terms with two

[2]Notice that the equations of motion are being used in order to reduce the number of operators. For instance, terms like $\langle P \, \nabla_\mu u^\mu \rangle$ are not present in eq. (2.4), since using eq. (B.1) (appendix B) we would generate operators that, either we have already considered, or with a higher number of resonances.

[3]This is much more clear in the case of vector and axial-vector resonances as their phenomenology is better known.

[4]This is in fact the origin of the subindex 2 in the lagrangian ($\mathcal{L}_2$), i.e. that the Goldstone and external currents are introduced with chiral tensors of that order.

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resonances that have the structure of the kinetic term, i.e. $\langle R\,R\,\chi^{(2)}\rangle$, as we have done[5] [12].

- The RχT lagrangian satisfies, by construction, the strictures of chiral dynamics at very low-energies ($E \ll M_R$). Notwithstanding, it is clear that there is no small coupling or kinematical parameter that could allow us to perform a perturbative expansion in order to solve the effective action of the theory, as it happens in χPT. Hence it looks that the exclusion of many (infinite) operators (e.g. one resonance and a $\mathcal{O}(p^4)$ chiral tensor, etc), satisfying the required symmetries, is not justified. The way out to this assessments has several components. On one side RχT is not an Effective Theory but a consistent phenomenological lagrangian model where many of the constraints from QCD are enforced. In particular it can be seen [12] that short-distance conditions limit strongly the operators that are allowed (those with higher order chiral tensors tend to violate the QCD ruled asymptotic behaviour of Green's functions or form factors). On the other side we have the large-N_C limit in order to guide a loop perturbative expansion, not in the lagrangian, but in the observables evaluated with it.

  It has also been proposed [22] that, due to the fact that the chiral counting is spoiled when resonances are included in loops, it could be possible to keep the chiral counting by disentangling the "*hard*" modes that could be absorbed in the renormalization program. In this way one gets a chiral expansion even if resonance contributions in the loop are considered. This procedure can be useful but only if one is interested in the application at very low energies out of the resonance region.

- The lagrangian theory described by eq. (2.1) satisfies the N_C counting rules [7, 23]. Leading operators in the large-N_C limit have one trace in the flavour space and we attach to this leading order terms. From the interaction vertices we see that the couplings F, c_d, c_m and d_m are of $\mathcal{O}(\sqrt{N_C})$ while $\lambda_i^{SS}, \lambda_i^{PP}$ and λ_i^{SP} are of $\mathcal{O}(1)$.

- Short-distance constraints on the asymptotic behaviour of form factors and Green functions [9, 13] provide, in the $N_C \to \infty$ limit, the following relations:

$$c_m = c_d = \sqrt{2}\, d_m = \frac{F}{2}\,, \tag{2.5}$$

  and $M_P^2 = 2\,(1-\delta)M_S^2$ with $\delta \simeq 3\,\pi\alpha_S\, F^2/M_S^2 \sim 0.08\,\alpha_S$. Short-distance constraints on the λ_i^{RR} couplings in the $N_C \to \infty$ are, at the moment, more controversial [12, 24, 25]:

$$\begin{aligned} \lambda_3^{SS} = \lambda_3^{PP} &= 0\,, \\ \lambda_1^{SP} = 4\,\lambda_2^{SP} &= -\frac{d_m}{c_m} = -\frac{1}{\sqrt{2}}\,, \end{aligned} \tag{2.6}$$

[5]Notice that field redefinitions could generate operators with chiral tensors of $\mathcal{O}(p^4)$ or larger in the chiral counting and would change the terms in the lagrangian of eq. (2.4). We choose a representation of the fields such that only operators with the structures $\langle\chi^{(2)}\rangle$, $\langle R\chi^{(2)}\rangle$ and $\langle RR\chi^{(2)}\rangle$ appear.

where we have used eq. (2.5). In appendix C we explain how these last results are obtained. Though the relations shown in eqs. (2.5), (2.6) could be used to simplify the outcome of the calculations presented in this article, we will give the full results without short-distance constraints built-in so as not to lose generality.

The lack of an expansion coupling or parameter in RχT hinders a perturbative renormalization like the one applied in χPT. Nevertheless a complete one-loop calculation of the vector form factor of the pion in RχT [11] was performed in ref. [20] with special attention to the renormalization program. It was shown that, using dimensional regularization, all the divergences could be absorbed by the introduction of local operators fulfilling the symmetry requirements. This is a particular case of the well known fact that all divergences are local in a quantum field theory [26], and are given by a polynomial in the external momenta or masses. Hence it is reasonable to consider the construction of the full set of operators that renders our $\mathcal{L}_{R\chi T}(\phi, S, P)$ theory finite up to one-loop. Accordingly we perform the one-loop generating functional of our lagrangian theory to evaluate the full set of divergences that arise. This we pursue in the rest of the article.

3. Generating functional at one loop

The generating functional of the connected Green functions, $W[J]$, is the logarithm of the vacuum-to-vacuum transition amplitude in the presence of external sources $J(x)$ coupled to bilinear quark currents:

$$e^{i\,W[J]} = \frac{1}{\mathcal{N}} \int [d\psi] e^{i\,S_0[\psi,J]} \,, \tag{3.1}$$

where the normalization is such that $W[0] = 0$ and the field ψ is, in our case, short for the Goldstone and resonance mesons. The evaluation of the generating functional of our lagrangian theory $\mathcal{L}_{R\chi T}(\phi, S, P)$, is readily done with the background field method [27, 28], where the action is expanded around the classical fields ψ_{cl}. By defining the quantum field as $\Delta\psi = \psi - \psi_{cl}$, the expansion up to one loop ($L = 1$) is given by:

$$\begin{aligned} W[J]_{L=1} = S_0[\psi_{cl}, J] - i \ln \Bigg[\int [d\Delta\psi] \exp \Bigg(i \int d^4x_1 \left. \frac{\delta\, S_0[\psi, J]}{\delta\psi_i(x_1)} \right|_{\psi_{cl}} \Delta\psi_i(x_1) \, + \\ + \frac{i}{2!} \int d^4x_1\, d^4x_2\, \Delta\psi_i(x_1) \, \times \\ \times \left. \frac{\delta^2\, S_0[\psi, J]}{\delta\psi_i(x_1)\, \delta\psi_j(x_2)} \right|_{\psi_{cl}} \Delta\psi_j(x_2) \Bigg) \Bigg] \,, \end{aligned} \tag{3.2}$$

but for an irrelevant constant. The i, j indices run over all the different fields and are summed over. The classical field ψ_{cl} is, by definition, the solution of:

$$\left. \frac{\delta S_0[\psi, J]}{\delta\psi_i(x)} \right|_{\psi_{cl}} = 0 \,, \tag{3.3}$$

that provides the implicit relation $\psi_{cl} = \psi_{cl}[J]$ and the Equations of Motion (EOM) for the classical fields. The explicit expressions of the latter are detailed in appendix B. Solving the remaining gaussian integral in the euclidean spacetime and coming back to Minkowsky we have finally:

$$W[J]_{L=1} \,=\, S_0[\psi_{cl}, J] + S_1[\psi_{cl}, J]\,, \tag{3.4}$$

$$S_1[\psi_{cl}, J] \,=\, \frac{i}{2}\ln\det\mathcal{D}(\psi_{cl}, J)\,, \tag{3.5}$$

where $\mathcal{D}(\psi_{cl}, J)$ is the quadratic differential operator specified by:

$$\langle\, x|\mathcal{D}(\psi_{cl}, J)|y\rangle_{ij} = \left.\frac{\delta^2\, S_0[\psi, J]}{\delta\psi_i(x)\,\delta\psi_j(y)}\right|_{\psi_{cl}}\,. \tag{3.6}$$

The action at one loop needs regularization and, following the use within χPT, we will proceed by working in D spacetime dimensions, a procedure that preserves the relevant symmetries of our theory. Divergences in the functional integration are local and, within dimensional regularization, can be absorbed through local operators that satisfy the same symmetries than the original theory [26]. The one-loop renormalized lagrangian is thus defined by:

$$\mathcal{L}_1[\psi, J] = \mu^{D-4}\left(\mathcal{L}_1^{\rm ren}[\psi, J;\mu] + \frac{1}{(4\pi)^2}\frac{1}{D-4}\mathcal{L}_1^{\rm div}[\psi, J]\right)\,. \tag{3.7}$$

In eq. (3.7) we have split the one-loop bare lagrangian into a renormalized and a divergent part, and the scale μ is introduced in order to restore the correct dimensions in the renormalized lagrangian for $D \neq 4$. The divergent part $\mathcal{L}_1^{\rm div}$ contains the counterterms which exactly cancel the divergences found in the result for the one-loop generating functional of eq. (3.4).

Up to one loop $\mathcal{L}_1[\psi, J]$ can be written in terms of a minimal basis of N operators $\mathcal{O}_i[\psi, J]$. For a non-renormalizable theory, such as RχT, N grows with the number of loops. Accordingly we expect to find in our evaluation of $S_1[\psi, J]$ many more operators that those in the original tree level theory $S_0[\psi, J]$. The structure of these obeys the same construction principles (symmetries) that gave $\mathcal{L}_{R\chi T}(\phi, S, P)$ in eq. (2.1), though we foresee that higher-order chiral tensors may be involved. A detailed study of the functional integration shows that the new terms have the structure $\chi^{(4)}$, $R\,\chi^{(4)}$ or $R\,R\,\chi^{(4)}$ (with a single or multiple traces) and $\chi^{(2)}$, $R\,\chi^{(2)}$ and $R\,R\,\chi^{(2)}$ (with multiple traces).[6]

3.1 Expansion around the classical solutions

Following the aforementioned procedure we expand the action associated to our lagrangian $\mathcal{L}_{R\chi T}(\phi, S, P)$ in eq. (2.1) around the solutions of the classical EOM: $u_{cl}(\phi)$, S_{cl} and P_{cl}. The fluctuations of the pseudoscalar Goldstone fields Δ_i ($i = 0, \ldots, 8$), and of the scalar and pseudoscalar resonances ε_{S_i} and ε_{P_i}, are parameterized as:[7]

$$u_R = u_{cl}\, e^{i\Delta/2}\,, \qquad u_L = u_{cl}^\dagger\, e^{-i\Delta/2}\,,$$

[6]As it will be emphasized later, in the procedure and due to a necessary field redefinition, terms with more than two resonances will be generated. We attach to our initial scheme and only will keep terms with up to two resonances.

[7]This is a convenient choice for the pseudoscalar fluctuation variables in order to simplify several cumbersome expressions. Notice that, once the "gauge" $u_R = u_L^\dagger \equiv u$ is enforced, it implies that the classical and the quantum Goldstone fields commute: $u_{cl}\exp(i\Delta/2) = \exp(i\Delta/2)u_{cl}$.

$$S = S_{cl} + \frac{1}{\sqrt{2}}\varepsilon_{\rm S}\,, \qquad P = P_{cl} + \frac{1}{\sqrt{2}}\varepsilon_{\rm P}\,, \tag{3.8}$$

with

$$\Delta = \frac{\Delta_i\lambda_i}{F}\,, \qquad \varepsilon_{\rm S} = \varepsilon_{{\rm S}i}\,\lambda_i\,, \qquad \varepsilon_{\rm P} = \varepsilon_{{\rm P}i}\,\lambda_i\,. \tag{3.9}$$

In the following we will drop the subindex "*cl*" for simplicity.

Expanding the lagrangian using eqs. (3.8), (3.9) up to terms quadratic in the fields (Δ_i, ε_{S_i}, ε_{P_i}) and using the EOM of appendix B, we obtain the second-order fluctuation lagrangian, that takes the form:[8]

$$\begin{aligned}\Delta\mathcal{L}_{{\rm R}\chi{\rm T}} = &-\frac{1}{2}\,\Delta_i\left(d'_\mu d'^\mu+\sigma\right)_{ij}\Delta_j - \frac{1}{2}\,\varepsilon_{{\rm S}i}\left(d^\mu d_\mu + k^{\rm S}\right)_{ij}\varepsilon_{{\rm S}j} - \\ &-\frac{1}{2}\,\varepsilon_{{\rm P}i}\left(d^\mu d_\mu + k^{\rm P}\right)_{ij}\varepsilon_{{\rm P}j} + \varepsilon_{{\rm S}i}\,{a^{\rm S}}_{ij}\,\Delta_j + \varepsilon_{{\rm P}i}\,{a^{\rm P}}_{ij}\,\Delta_j + \varepsilon_{{\rm P}i}\,{a^{\rm SP}}_{ij}\,\varepsilon_{{\rm S}j} + \\ &+\varepsilon_{{\rm S}k}\,b^{\rm S}_{\mu\,ki}\,d^\mu_{ij}\Delta_j + \varepsilon_{{\rm P}k}\,b^{\rm P}_{\mu\,ki}\,d^\mu_{ij}\Delta_j + \varepsilon_{{\rm P}k}\,b^{\rm SP}_{\mu\,ki}\,d^\mu_{ij}\varepsilon_{{\rm S}j}\,. \end{aligned}\tag{3.10}$$

Derivatives and matrices are defined in appendix D where it is also shown that in order to write $\Delta\mathcal{L}_{R\chi T}$ in the form displayed above we need to perform two field redefinitions. This procedure generates operators with multiple resonance fields. However our theory, as specified in section 2, does not include operators with more than two resonances and, for consistency, we shall keep this structure in the fluctuation lagrangian, thus disregarding operators with three or more resonance fields in the following. We will comment later on the consequences of this feature. It is customary to write the second-order fluctuation lagrangian as:

$$\Delta\mathcal{L}_{{\rm R}\chi{\rm T}} = -\frac{1}{2}\,\eta\left(\Sigma_\mu\,\Sigma^\mu + \Lambda\right)\eta^\top\,, \tag{3.11}$$

where η collects the fluctuation fields, $\eta = (\Delta_i, \varepsilon_{{\rm S}j}, \varepsilon_{{\rm P}k})$, $i,j,k = 0,\ldots,8$, $\eta^\top$ is its transposed and the rest of definitions are given in appendix D.

3.2 Divergent part of the generating functional at one loop

After we have performed the second-order fluctuation on our lagrangian theory we come back to our discussion at the beginning of this section in order to identify the one-loop generating functional, specified now by the action:

$$S_1 = \frac{i}{2}\ln\det\left(\Sigma_\mu\,\Sigma^\mu + \Lambda\right). \tag{3.12}$$

We use dimensional regularization to extract the divergence of this expression. As emphasized in the literature [29] it is convenient to employ the Schwinger-DeWitt proper-time representation, embedded in the heat-kernel formalism, in order to extract the residue at the $D-4$ pole. Ref. [27] shows that, in fact, symmetry considerations can also provide this information (at least up to one loop).

Hence we get:

$$S_1 = -\frac{1}{(4\pi)^2}\frac{1}{D-4}\int {\rm d}^4x\,{\rm Tr}\left(\frac{1}{12}\,Y_{\mu\nu}\,Y^{\mu\nu} + \frac{1}{2}\Lambda^2\right) + S_1^{\rm finite}\,, \tag{3.13}$$

[8] The intricacies of this evaluation are explained in detail in appendix D.

where Tr is short for the trace in the flavour space, $Y_{\mu\nu}$ denotes the field strength tensor of Y_μ in eq. (D.26): $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu + [Y_\mu, Y_\nu]$. The finite remainder S_1^{finite} cannot be simply expressed as a local lagrangian, but can be worked out for a given transition [2, 5].

Finally we get the one-loop divergence as:

$$S_1^{\text{div}} = -\frac{1}{(4\,\pi)^2}\frac{1}{D-4}\int d^4x\,\mathcal{L}_1^{\text{div}}\,, \tag{3.14}$$

where

$$\begin{aligned}
\mathcal{L}_1^{\text{div}} = {} & \frac{1}{12}\langle \gamma'_{\mu\nu}\gamma'^{\,\mu\nu} + 2\gamma_{\mu\nu}\gamma^{\mu\nu}\rangle + \frac{1}{2}\langle \sigma^2 + k^{\text{P}\,2} + k^{\text{S}\,2}\rangle + \\
& + \langle a^{\text{S}} a^{\text{S}\,\top} + a^{\text{P}} a^{\text{P}\,\top} + a^{\text{SP}} a^{\text{SP}\,\top}\rangle - \frac{1}{12}\left\langle \gamma'^{\,\mu\nu}\left(b_\mu^{\text{S}\,\top} b_\nu^{\text{S}} + b_\mu^{\text{P}\,\top} b_\nu^{\text{P}}\right)\right\rangle - \\
& - \frac{1}{12}\left\langle \gamma^{\mu\nu}\left(b_\mu^{\text{S}} b_\nu^{\text{S}\,\top} + b_\mu^{\text{P}} b_\nu^{\text{P}\,\top} + b_\mu^{\text{SP}} b_\nu^{\text{SP}\,\top} + b_\mu^{\text{SP}\,\top} b_\nu^{\text{SP}}\right)\right\rangle - \\
& - \left\langle a^{\text{S}\,\top}\left(\bar{d}_+^\mu b_\mu^{\text{S}} + \frac{1}{2} b_\mu^{\text{SP}\,\top} b^{\text{P}\,\mu}\right) + a^{\text{P}\,\top}\left(\bar{d}_+^\mu b_\mu^{\text{P}} - \frac{1}{2} b_\mu^{\text{SP}} b^{\text{S}\,\mu}\right) + \right. \\
& \quad \left. + a^{\text{SP}\,\top}\left(\hat{d}^\mu b_\mu^{\text{SP}} + \frac{1}{2} b^{\text{P}\,\mu} b_\mu^{\text{S}\,\top}\right)\right\rangle + \\
& + \frac{1}{4}\left\langle \sigma\left(b_\mu^{\text{S}\,\top} b^{\text{S}\,\mu} + b_\mu^{\text{P}\,\top} b^{\text{P}\,\mu}\right) + k^{\text{S}}\left(b^{\text{S}\,\mu} b_\mu^{\text{S}\,\top} + b_\mu^{\text{SP}\,\top} b^{\text{SP}\,\mu}\right) + \right. \\
& \quad \left. + k^{\text{P}}\left(b^{\text{P}\,\mu} b_\mu^{\text{P}\,\top} + b^{\text{SP}\,\mu} b_\mu^{\text{SP}\,\top}\right)\right\rangle + \\
& + \frac{1}{4}\langle \tilde{d}_-^\mu b_\mu^{\text{S}\,\top} \bar{d}_+^\nu b_\nu^{\text{S}} + \tilde{d}_-^\mu b_\mu^{\text{P}\,\top} \bar{d}_+^\nu b_\nu^{\text{P}} + \hat{d}^\mu b_\mu^{\text{SP}\,\top} \hat{d}^\nu b_\nu^{\text{SP}}\rangle - \\
& - \frac{1}{12}\langle \tilde{d}_{+\mu} b_\nu^{\text{S}\,\top} \bar{d}_-^{[\mu} b^{\text{S}\,\nu]} + \tilde{d}_{+\mu} b_\nu^{\text{P}\,\top} \bar{d}_-^{[\mu} b^{\text{P}\,\nu]} + \hat{d}_\mu b_\nu^{\text{SP}\,\top} \hat{d}^{[\mu} b^{\text{SP}\,\nu]}\rangle + \\
& + \frac{1}{4}\langle \tilde{d}_-^\mu b_\mu^{\text{S}\,\top} b_\nu^{\text{SP}\,\top} b^{\text{P}\,\nu} - \tilde{d}_-^\mu b_\mu^{\text{P}\,\top} b_\nu^{\text{SP}} b^{\text{S}\,\nu} + \hat{d}^\mu b_\mu^{\text{SP}\,\top} b^{\text{P}\,\nu} b_\nu^{\text{S}\,\top}\rangle - \\
& - \frac{1}{12}\langle \tilde{d}_+^\mu b^{\text{S}\,\nu\,\top} b_{[\mu}^{\text{SP}\,\top} b_{\nu]}^{\text{P}} - \tilde{d}_+^\mu b^{\text{P}\,\nu\,\top} b_{[\mu}^{\text{SP}} b_{\nu]}^{\text{S}} + \hat{d}^\mu b^{\text{SP}\,\nu\,\top} b_{[\mu}^{\text{P}} b_{\nu]}^{\text{S}\,\top}\rangle + \\
& + \frac{1}{48}\left\langle \left(b_\mu^{\text{S}\,\top} b^{\text{S}\,\mu} b_\nu^{\text{S}\,\top} b^{\text{S}\,\nu} + b_\mu^{\text{S}\,\top} b^{\text{S}\,\nu} b_\nu^{\text{S}\,\top} b^{\text{S}\,\mu} + b_\mu^{\text{S}\,\top} b_\nu^{\text{S}} b^{\text{S}\,\mu\,\top} b^{\text{S}\,\nu}\right) + \right. \\
& \quad + \left(b_\mu^{\text{P}\,\top} b^{\text{P}\,\mu} b_\nu^{\text{P}\,\top} b^{\text{P}\,\nu} + b_\mu^{\text{P}\,\top} b^{\text{P}\,\nu} b_\nu^{\text{P}\,\top} b^{\text{P}\,\mu} + b_\mu^{\text{P}\,\top} b_\nu^{\text{P}} b^{\text{P}\,\mu\,\top} b^{\text{P}\,\nu}\right) + \\
& \quad \left. + \left(b_\mu^{\text{SP}\,\top} b^{\text{SP}\,\mu} b_\nu^{\text{SP}\,\top} b^{\text{SP}\,\nu} + b_\mu^{\text{SP}\,\top} b^{\text{SP}\,\nu} b_\nu^{\text{SP}\,\top} b^{\text{SP}\,\mu} + b_\mu^{\text{SP}\,\top} b_\nu^{\text{SP}} b^{\text{SP}\,\mu\,\top} b^{\text{SP}\,\nu}\right)\right\rangle + \\
& + \frac{1}{24}\left\langle \left(b_\mu^{\text{S}\,\top} b^{\text{S}\,\mu} b_\nu^{\text{P}\,\top} b^{\text{P}\,\nu} + b_\mu^{\text{S}\,\top} b^{\text{S}\,\nu} b_\nu^{\text{P}\,\top} b^{\text{P}\,\mu} + b_\mu^{\text{S}\,\top} b_\nu^{\text{S}} b^{\text{P}\,\mu\,\top} b^{\text{P}\,\nu}\right) + \right. \\
& \quad + \left(b_\mu^{\text{SP}\,\top} b^{\text{SP}\,\mu} b^{\text{S}\,\nu} b_\nu^{\text{S}\,\top} + b_\mu^{\text{SP}\,\top} b^{\text{SP}\,\nu} b_\nu^{\text{S}} b^{\text{S}\,\mu\,\top} + b_\mu^{\text{SP}\,\top} b^{\text{SP}\,\nu} b^{\text{S}\,\mu} b_\nu^{\text{S}\,\top}\right) + \\
& \quad \left. + \left(b^{\text{P}\,\mu} b_\mu^{\text{P}\,\top} b^{\text{SP}\,\nu} b_\nu^{\text{SP}\,\top} + b^{\text{P}\,\mu} b_\nu^{\text{P}\,\top} b^{\text{SP}\,\nu} b_\mu^{\text{SP}\,\top} + b_\mu^{\text{P}} b_\nu^{\text{P}\,\top} b^{\text{SP}\,\mu} b^{\text{SP}\,\nu\,\top}\right)\right\rangle ,
\end{aligned} \tag{3.15}$$

where derivatives and matrices are defined in appendix D and $\gamma_{\mu\nu} = \partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu + [\gamma_\mu, \gamma_\nu]$ (correspondingly for $\gamma'_{\mu\nu}$). Moreover for two vectors A_μ,B_μ we write $A_{[\mu}B_{\nu]} = A_\mu B_\nu - A_\nu B_\mu$. This result is completely general for the second-order fluctuation lagrangian in eq. (3.10). However, and as explained in appendix D, the expressions given there are valid only for operators with up to two resonances as we limit ourselves in this article.

3.3 Result

When worked out, $S_1^{\rm div}$ in eq. (3.14) can be expressed in a basis of operators that satisfy the same symmetry requirements than our starting lagrangian $\mathcal{L}_{\rm R\chi T}(\phi, S, P)$. A minimal basis of $R\chi T$ operators that, upon integration of the resonances, contributes to the $\mathcal{O}(p^6)$ χPT lagrangian, in SU(3), will be found elsewhere [12]. However, up to now, a basis for the one-loop $R\chi T$ has still not been worked out. This is precisely our result generated by $S_1^{\rm div}$. Hence, at one loop, the $R\chi T$ lagrangian needed to renormalize our theory reads:

$$\mathcal{L}_1 = \sum_{i=1}^{18} \alpha_i\, \mathcal{O}_i + \sum_{i=1}^{68} \beta_i^R\, \mathcal{O}_i^R + \sum_{i=1}^{383} \beta_i^{RR}\, \mathcal{O}_i^{RR} \,. \tag{3.16}$$

The $\mathcal{O}_i$ operators correspond to those up to $\mathcal{O}(p^4)$ in $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ χPT [30]. $\mathcal{O}_i^R$ and $\mathcal{O}_i^{RR}$ involve one and two resonance fields, respectively, together with $\chi^{(2)}$ and $\chi^{(4)}$ chiral tensors. The couplings in the bare lagrangian $\mathcal{L}_1$ read, in accordance with eq. (3.7):

$$\begin{aligned}
\alpha_i &= \mu^{D-4}\left(\alpha_i^r(\mu) + \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\gamma_i\right), \\
\beta_i^R &= \mu^{D-4}\left(\beta_i^{R,r}(\mu) + \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\gamma_i^R\right), \\
\beta_i^{RR} &= \mu^{D-4}\left(\beta_i^{RR,r}(\mu) + \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\gamma_i^{RR}\right),
\end{aligned} \tag{3.17}$$

where γ_i, γ_i^R and γ_i^{RR} are the divergent coefficients given by $S_1^{\rm div}$ that constitute the β-function of our lagrangian (we use the terminology of ref. [21]). The determination of the latter though straightforward involves a long calculation. In order to diminish the possibility of errors we have performed two independent evaluations. One of them has been carried out with the help of the FORM 3 program [31] and the other with Mathematica [32]. The full result when scalar and pseudoscalar resonances are included is rather lengthy. In appendix E we provide the sets $\mathcal{O}_i$, $\mathcal{O}_i^S$ and $\mathcal{O}_i^{SS}$. The coefficients γ_i are fully given, however γ_i^S and γ_i^{SS} are only brought when just scalar resonances are considered, which reduces the total number of operators to 177. The complete result with the inclusion of pseudoscalar resonances is available at `http://ific.uv.es/quiral/rt1loop.html` or upon request from the authors.

4. Features and use of the renormalized RχT lagrangian

In order to understand the aspects and use of the renormalized $R\chi T$ lagrangian that we have obtained above, we would like to emphasize here several of its features:

1. In table 1 we have collected the full basis of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ χPT operators generated in the functional integration at one loop. We should recover the result first obtained in ref. [30]. After the comparison is made[9] we agree indeed with

[9]Notice that the notation of ref. [30] is different to ours though, to ease the comparison, the order chosen is the same. We always quote our notation for the operators.

their results. Notice though that in order to disentangle the resonances, it is not enough to withdraw all the resonance couplings. This is because the derivative terms in $\mathcal{L}_{\rm kin}(S,P)$, which do not carry any resonance coupling, also contribute through the functional integration to several of the operators, namely $\mathcal{O}_4$, $\mathcal{O}_7$, $\mathcal{O}_{13}$, $\mathcal{O}_{14}$ and $\mathcal{O}_{15}$ in table 1. We have confirmed that $\mathcal{L}_{\rm kin}(S,P)$ gives precisely the difference between our coefficients γ_4, γ_7, γ_{13}, γ_{14}, γ_{15} and those of ref. [30] once the resonance couplings have been switched off.

2. Our result provides the running of the α_i, β_i^R and β_i^{RR} couplings through the renormalization group equations (RGE). From eq. (3.17) we get:

$$\mu \frac{d}{d\mu}\, \alpha_i^r(\mu) = -\frac{\gamma_i}{16\,\pi^2}\,, \tag{4.1}$$

and, analogously, for β_i^R and β_i^{RR}. This result can be potentially useful if we are interested in the phenomenological evaluation of the resonance couplings at this order. Though μ is known to be of the order of a typical scale of the physical system, let us say $\mu = M_S$ or $\mu = M_P$, there always remains some ambiguity on the precise value of μ at which the low-energy couplings are extracted from the phenomenology. The RGE (4.1) provides an estimate of the reliance of such determinations. If the coupling under request varies drastically with the scale it is clear that the value obtained phenomenologically has a large uncertainty, while if it has a smooth running the determination is more reliable.

Within this issue it is interesting to take a closer look to the running of the resonance couplings in the original RχT lagrangian [11], namely, $c_d^r(\mu)$, $c_m^r(\mu)$ and $d_m^r(\mu)$. Their corresponding β-function coefficients are β_1^S, β_4^S and β_{44}^P, respectively (notice that we are taking into account here the complete results, including scalar and pseudoscalar resonances). We obtain:

$$\begin{aligned}
\mu \frac{d}{d\mu}\, c_d^r(\mu) &= -\frac{N}{16\pi^2}\, c_d \frac{M_S^2}{F^2}\left[2\lambda_1^{SS} + 4\lambda_2^{SS} - 3\frac{c_d^2}{F^2} - (\lambda_1^{SP})^2\right], \\
\mu \frac{d}{d\mu}\, c_m^r(\mu) &= -\frac{N}{32\pi^2}\, \frac{M_S^2}{F^2}\left[c_d\left(1 - 4\frac{c_d c_m}{F^2}\right) - 4\lambda_1^{SP}\left(d_m + c_m\lambda_1^{SP}\right)\right], \\
\mu \frac{d}{d\mu}\, d_m^r(\mu) &= -\frac{N}{16\pi^2}\, \lambda_1^{SP}\, \frac{M_P^2}{F^2}\left(1 - 2\frac{M_S^2}{M_P^2}\right)\left[c_d - 2\,c_m - 2\,d_m\lambda_1^{SP}\right],
\end{aligned} \tag{4.2}$$

where N is the number of flavours. First of all note that the running, as expected, is a next-to-leading order effect in the $1/N_C$ expansion. This fact is guided by the M_R^2/F^2 factors taking into account that $M_R \sim \mathcal{O}(1)$ and $F \sim \mathcal{O}(\sqrt{N_C})$. Another interesting aspect is the interval over which μ runs. It is well known [26] that the couplings are only relevant at the scale of the momenta involved in the processes (in order to diminish the role of the logarithms). In our case $\mu \sim M_S$, M_P. Thus we do not expect a large running for the scale, namely a few hundreds of MeV. This last conclusion brings us to the next point. At next to leading order in $1/N_C$ we can ignore the running on the right-hand-side of eqs. (4.2). Hence we can input the leading order values for the couplings, given by eqs. (2.5), (2.6) and obtained from the short-distance constraints in section 2, to get the leading logarithm in the evolution of $c_m^r(\mu)$ and $d_m^r(\mu)$. It is remarkable that, at this order, eqs. (4.2) predict no running for these coupling constants, as the right-hand-side of their RGE vanishes. Unfortunately we cannot conclude anything about c_d, as there are no known constraints on λ_1^{SS} and λ_2^{SS}. If the Large-N_C estimates for the couplings are to be reliable we come to the conclusion that the predictions for c_m and d_m are rather robust. Moreover the same exercise for the $\mathcal{O}_i$ operators involving only Goldstone and external fields in table 1 shows that also $\mathcal{O}_2$, $\mathcal{O}_8$, $\mathcal{O}_9$, $\mathcal{O}_{10}$, $\mathcal{O}_{11}$, $\mathcal{O}_{12}$, $\mathcal{O}_{16}$ and $\mathcal{O}_{18}$ do not run at one loop, i.e. all those operators involving χ_+ and/or χ_-. Also notice that c_m and d_m rule operators sharing this feature.

3. In the procedure we have employed to evaluate the functional integration of $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ up to one loop we have withdrawn those operators with three or more resonance fields and kept up to two resonances. A cut in the number of resonances is necessary because to reach the gaussian expression in eq. (3.11) we need to perform several field transformations (see appendix D) that generate operators with more resonance fields which in turn require additional field transformations and so on. One of the differences of RχT with respect to χPT (in the strong [2] or electroweak interaction [33] form of the latter) is that we do not have an expansion parameter into the lagrangian that can provide a natural cut for higher order terms in these field transformations. Notice that the cut in the number of resonances seems to hinder our result, as it does not allow us to renormalize divergent one loop diagrams with three or more resonance fields as external legs. However we would not expect to treat these loops as we are not including, in our leading order lagrangian, interacting terms with three or more resonance fields.

To end this section we would like to show a simple example of the application of our result. We consider the one-loop renormalization of the two-point function of scalar currents:

$$\Pi_{SS}^{ij}(q^2) = i \int d^4x \, e^{iq\cdot x} \, \langle 0|T\{S^i(x) S^j(0)\}|0\rangle \,, \qquad S^i(x) = \overline{q}(x)\lambda^i q(x)\,, \tag{4.3}$$

in the chiral limit and when only scalar resonances are considered. The divergent loop diagrams contributing are those depicted in figure 1. In order to cancel the divergences one needs to add the counterterm contributions in figure 2, where diagram C_1 is given by $\mathcal{O}_{12} + 2\mathcal{O}_{16} = \langle \chi_+^2 \rangle$ in table 1, C_2 by $\mathcal{O}_4^S = \langle S\chi_+ \rangle$ in table 2 and C_3 by $\mathcal{O}_1^{SS} = \langle SS \rangle$ in table 3. The cancellation works as follows: One part of the contribution of C_1 cancels completely the divergence in the loops $L_1 + L_2$. Another piece of C_1 together with C_2 eliminates the divergence coming from L_3 and, finally, all remaining contributions of C_1 and C_2 add to C_3 in order to render L_4 finite. Notice that, as there are no nonlocal divergences, the contributions of 1PR diagrams are brought finite once 1PI diagrams have been properly renormalized.

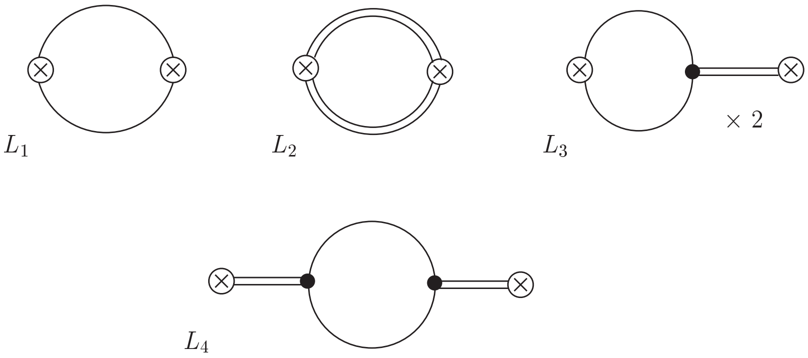

Figure 1: One-loop contributions to the $\Pi_{SS}^{ij}(q^2)$ correlator in the chiral limit when only scalar resonances are included. A single line stands for a Goldstone boson while a double line indicates a scalar resonance. Their result is divergent.

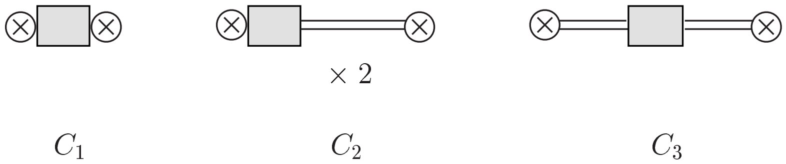

Figure 2: Counterterm contributions that renormalize the one-loop result of figure 1. A double line stands for a scalar resonance.

5. Summary

RχT provides a consistent framework to study the energy region of the hadronic resonances, $M_V \lesssim E \lesssim 2$ GeV. It embodies a phenomenological lagrangian where Goldstone bosons and resonances fields are kept as active degrees of freedom; this is the key ingredient for the application of the large-N_C expansion. Recently, and after its multiple explorations at tree level, it has emerged some interest in the application of RχT at one loop level mainly to understand how the features of QCD are implemented into the theory.

In this work we have systematically obtained, by using the background field method and for the first time, both the full basis of operators and the β-function coefficients that render finite, up to one loop, our initial lagrangian $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ in eq. (2.1). This would correspond to the next-to-leading order in the $1/N_C$ expansion but including one multiplet of scalar and pseudoscalar resonances only. Our main result is given by eq. (3.16) and the γ_i, γ_i^R, γ_i^{RR} parameters in eq. (3.17). The outcome is relevant for the study of those diagrams involving a loop with up to two resonances and any number of Goldstone bosons in the legs.

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A. Notation

In this article we follow the notation introduced in ref. [11] that we collect here.

The non-linear realization of spontaneously broken chiral symmetry $G = \mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ is defined by the action of the group on the elements $u_{R,L}(\phi)$ of the coset space $G/\,\mathrm{U}(3)_V$:

$$\begin{aligned} u_L(\phi) &\xrightarrow{G} g_L\, u_L(\phi)\, h(g,\phi)^\dagger \,, \\ u_R(\phi) &\xrightarrow{G} g_R\, u_R(\phi)\, h(g,\phi)^\dagger \,, \end{aligned} \tag{A.1}$$

where $g = (g_L, g_R) \in G$ and $h(g,\phi) \in \mathrm{U}(3)_V$.

The following chiral tensors have been used:

$$\begin{aligned} u_\mu &= i\left\{ u_R^\dagger\,(\partial_\mu - i r_\mu)\, u_R - u_L^\dagger\,(\partial_\mu - i\ell_\mu)\, u_L \right\}, \\ \Gamma_\mu &= \frac{1}{2}\left\{ u_R^\dagger\,(\partial_\mu - i r_\mu)\, u_R + u_L^\dagger\,(\partial_\mu - i\ell_\mu)\, u_L \right\}, \\ \chi_\pm &= u_R^\dagger \chi u_L \pm u_L^\dagger \chi^\dagger u_R \,, \\ \chi &= 2B_0\,(s + ip)\,, \end{aligned} \tag{A.2}$$

where s, p, r and ℓ are scalar, pseudoscalar, right and left external fields, respectively. The covariant derivative is defined by

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X]\,, \tag{A.3}$$

for any X transforming as

$$X \xrightarrow{G} h(g,\phi)\, X\, h(g,\phi)^\dagger \,, \tag{A.4}$$

under the chiral group, like the tensors u_μ, $\chi_\pm$ or the resonance fields S, P.

We can take the choice of coset representative such that $u_R(\phi) = u_L^\dagger(\phi) \equiv u(\phi)$, whose explicit form in the exponential parameterization is:

$$u(\phi) = \exp\left(\frac{i}{\sqrt{2}\,F}\,\phi \right), \qquad \phi = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i\, \phi_i \,, \tag{A.5}$$

where the normalization of the Gell-Mann matrices is given by $\langle \lambda_i \lambda_j \rangle = 2\delta_{ij}$ and ϕ is the nonet of pseudo-Goldstone fields. Scalar and pseudoscalar resonances have analogous $\mathrm{U}(3)_V$ content to the one described by ϕ:

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}. \tag{A.6}$$

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B. Equations of motion for the classical fields

The classical fields are defined by eq. (3.3). From the RχT lagrangian in eq. (2.1), the EOM's for the Goldstone and resonance fields are obtained as the system of coupled equations:

$$
\begin{aligned}
\nabla^\mu u_\mu =\; & \frac{i}{2}\chi_- - \frac{2c_d}{F^2}\nabla^\mu\{u_\mu, S\} + \frac{i\,c_m}{F^2}\{\chi_-, S\} - \frac{1}{2F^2}[u_\mu, [\nabla^\mu S, S]] - \\
& -\frac{2\lambda_1^{\rm SS}}{F^2}\nabla_\mu\{u^\mu, SS\} - \frac{4\lambda_2^{\rm SS}}{F^2}\nabla_\mu(S\,u^\mu S) + \frac{i\,\lambda_3^{\rm SS}}{F^2}\{\chi_-, SS\} - \\
& -\frac{d_m}{F^2}\{\chi_+, P\} - \frac{1}{2F^2}[u_\mu, [\nabla^\mu P, P]] - \frac{2\lambda_1^{\rm PP}}{F^2}\nabla_\mu\{u^\mu, PP\} - \\
& -\frac{4\lambda_2^{\rm PP}}{F^2}\nabla_\mu(P\,u^\mu P) + \frac{i\,\lambda_3^{\rm PP}}{F^2}\{\chi_-, PP\} - \frac{2\lambda_1^{\rm SP}}{F^2}\nabla^\mu\{\nabla_\mu S, P\} + \\
& +\frac{\lambda_1^{\rm SP}}{2F^2}\Big[u_\mu, \big[S, \{P, u^\mu\}\big]\Big] - \frac{\lambda_2^{\rm SP}}{F^2}\{\chi_+, \{S, P\}\}\,, \qquad \text{(B.1)}\\
\nabla^\mu\nabla_\mu S =\; & -M_S^2\,S + c_m\,\chi_+ + c_d\,u_\mu u^\mu + \lambda_1^{\rm SS}\{S, u_\mu u^\mu\} + 2\lambda_2^{\rm SS}u_\mu S u^\mu + \\
& +\lambda_3^{\rm SS}\{S, \chi_+\} - \lambda_1^{\rm SP}\nabla_\mu\{P, u^\mu\} + i\lambda_2^{\rm SP}\{P, \chi_-\}\,, \\
\nabla^\mu\nabla_\mu P =\; & -M_P^2\,P + i\,d_m\,\chi_- + \lambda_1^{\rm PP}\{P, u_\mu u^\mu\} + 2\lambda_2^{\rm PP}u_\mu P u^\mu + \\
& +\lambda_3^{\rm PP}\{P, \chi_+\} + \lambda_1^{\rm SP}\{\nabla_\mu S, u^\mu\} + i\lambda_2^{\rm SP}\{S, \chi_-\}\,. \qquad \text{(B.2)}
\end{aligned}
$$

C. Short-distance constraints on the λ_i^{RR} couplings

Most of the short-distance constraints on the couplings of RχT come from matching Green functions of QCD currents evaluated within the resonance theory with the results obtained in the leading OPE expansion [15]. Another source of conditions arise from form factors. Parton dynamics demands that two-body form factors of hadronic currents should vanish at infinite momentum transfer [34], property known as Brodsky-Lepage behaviour. It is also phenomenologically well known that any hadronic form factor seems to share this feature, at least in the case of stable asymptotic states.

Form factors of QCD currents that involve resonance asymptotic states do not necessarily fall in the group above. Hence it is not clear whether the latter should satisfy the Brodsky-Lepage behaviour. It is often claimed that as resonances are non-decaying states in the $N_C \to \infty$ limit, they should not be treated differently from (real) stable states. However one shall keep in mind that the strict large-N_C limit also requires to consider an infinite number of resonances, so the discussion is far from settled. Within this discussion it has been also observed [25] that not always all short-distance constraints can be satisfied at the same time.

Leaving aside this issue, if one enforces the Brodsky-Lepage behaviour on the resonance form factors useful constraints among the couplings are found [12]. From the pseudoscalar form factor $\langle P^i|p^j|S^k\rangle$ and $\langle P^i|s^j|\pi^k\rangle$ we obtain, respectively,

$$\lambda_1^{SP} = 4\,\lambda_2^{SP}\,, \qquad \text{(C.1)}$$

and

$$\lambda_1^{SP} = -\frac{d_m}{c_m}\,. \tag{C.2}$$

Finally the scalar form factors $\langle P^i|s^j|P^k\rangle$ and $\langle S^i|s^j|S^k\rangle$ give

$$\lambda_3^{PP} = \lambda_3^{SS} = 0\,. \tag{C.3}$$

D. Second-order fluctuation of the lagrangian

The expansion around the classical solution of the fields in our lagrangian $\mathcal{L}_{R\chi T}(\phi, S, P)$ up to second order (as required for the one loop evaluation) gives:

$$\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \Delta\mathcal{L}_\chi^{(2)} + \Delta\mathcal{L}_{\mathrm{kin}}(\mathrm{S},\mathrm{P}) + \Delta\mathcal{L}_2(\mathrm{S}) + \Delta\mathcal{L}_2(\mathrm{P}) + \Delta\mathcal{L}_2(\mathrm{S},\mathrm{P})\,, \tag{D.1}$$

where

$$\Delta\mathcal{L}_\chi^{(2)} = -\frac{F^2}{8}\left\langle\chi_+\Delta^2\right\rangle + \frac{F^2}{4}\left\langle\nabla^\mu\Delta\nabla_\mu\Delta + \frac{1}{4}\,[u_\mu,\Delta]\,[u^\mu,\Delta]\right\rangle, \tag{D.2}$$

$$\begin{aligned}
\Delta\mathcal{L}_{\mathrm{kin}}(\mathrm{S},\mathrm{P}) = {} & \frac{1}{4}\left\langle\nabla^\mu\varepsilon_{\mathrm{S}}\nabla_\mu\varepsilon_{\mathrm{S}}\right\rangle - \frac{M_S^2}{4}\left\langle\varepsilon_{\mathrm{S}}\,\varepsilon_{\mathrm{S}}\right\rangle + \frac{1}{32}\left\langle\big[[u^\mu,\Delta],S\big]\big[[u_\mu,\Delta],S\big]\right\rangle - \\
& -\frac{1}{8}\left\langle[\nabla_\mu\Delta,\Delta][S,\nabla^\mu S]\right\rangle + \frac{1}{4\sqrt{2}}\left\langle[u_\mu,\Delta]\Big([S,\nabla^\mu\varepsilon_{\mathrm{S}}] - [\nabla^\mu S,\varepsilon_{\mathrm{S}}]\Big)\right\rangle + \\
& +\frac{1}{4}\left\langle\nabla^\mu\varepsilon_{\mathrm{P}}\nabla_\mu\varepsilon_{\mathrm{P}}\right\rangle - \frac{M_P^2}{4}\left\langle\varepsilon_{\mathrm{P}}\,\varepsilon_{\mathrm{P}}\right\rangle + \frac{1}{32}\left\langle\big[[u^\mu,\Delta],P\big]\big[[u_\mu,\Delta],P\big]\right\rangle - \\
& -\frac{1}{8}\left\langle[\nabla_\mu\Delta,\Delta][P,\nabla^\mu P]\right\rangle + \frac{1}{4\sqrt{2}}\left\langle[u_\mu,\Delta]\Big([P,\nabla^\mu\varepsilon_{\mathrm{P}}] - [\nabla^\mu P,\varepsilon_{\mathrm{P}}]\Big)\right\rangle, \qquad \text{(D.3)}\\
\Delta\mathcal{L}_2(\mathrm{S}) = {} & -\frac{i\,c_m}{2\sqrt{2}}\left\langle\varepsilon_{\mathrm{S}}\{\Delta,\chi_-\}\right\rangle - \frac{c_m}{8}\left\langle\{S,\Delta\}\{\chi_+,\Delta\}\right\rangle - \frac{c_d}{\sqrt{2}}\left\langle\varepsilon_{\mathrm{S}}\{\nabla_\mu\Delta,u^\mu\}\right\rangle + \\
& +\left\langle\Big(c_d\,S + \lambda_1^{\mathrm{SS}}SS\Big)\left(\nabla^\mu\Delta\nabla_\mu\Delta + \frac{1}{8}\Big\{\big[\Delta,[u_\mu,\Delta]\big],u^\mu\Big\}\right)\right\rangle + \\
& +\frac{\lambda_1^{\mathrm{SS}}}{2}\left\langle\varepsilon_{\mathrm{S}}^2\,u^\mu u_\mu\right\rangle - \frac{\lambda_1^{\mathrm{SS}}}{\sqrt{2}}\left\langle\{S,\varepsilon_{\mathrm{S}}\}\,\{u_\mu,\nabla^\mu\Delta\}\right\rangle + \\
& +\lambda_2^{\mathrm{SS}}\left\langle S\,\nabla_\mu\Delta\,S\,\nabla^\mu\Delta\right\rangle + \frac{\lambda_2^{\mathrm{SS}}}{2}\left\langle\varepsilon_{\mathrm{S}}u_\mu\varepsilon_{\mathrm{S}}u^\mu\right\rangle - \\
& -\sqrt{2}\lambda_2^{\mathrm{SS}}\left\langle\varepsilon_{\mathrm{S}}\big(\nabla_\mu\Delta\,S\,u^\mu + u_\mu S\,\nabla^\mu\Delta\big)\right\rangle + \frac{\lambda_2^{\mathrm{SS}}}{4}\left\langle\big[[\Delta,u_\mu],\Delta\big]S\,u^\mu S\right\rangle - \\
& -\frac{\lambda_3^{\mathrm{SS}}}{8}\left\langle\{SS,\Delta\}\{\chi_+,\Delta\}\right\rangle - \frac{i\,\lambda_3^{\mathrm{SS}}}{2\sqrt{2}}\left\langle\{S,\varepsilon_{\mathrm{S}}\}\{\chi_-,\Delta\}\right\rangle + \frac{\lambda_3^{\mathrm{SS}}}{2}\left\langle\varepsilon_{\mathrm{S}}^2\,\chi_+\right\rangle, \qquad \text{(D.4)}\\
\Delta\mathcal{L}_2(\mathrm{P}) = {} & \frac{d_m}{2\sqrt{2}}\left\langle\varepsilon_{\mathrm{P}}\{\Delta,\chi_+\}\right\rangle - \frac{i\,d_m}{8}\left\langle\{P,\Delta\}\{\chi_-,\Delta\}\right\rangle + \\
& +\lambda_1^{\mathrm{PP}}\left\langle PP\left(\nabla^\mu\Delta\nabla_\mu\Delta + \frac{1}{8}\Big\{\big[\Delta,[u_\mu,\Delta]\big],u^\mu\Big\}\right)\right\rangle + \frac{\lambda_1^{\mathrm{PP}}}{2}\left\langle\varepsilon_{\mathrm{P}}^2\,u^\mu u_\mu\right\rangle - \\
& -\frac{\lambda_1^{\mathrm{PP}}}{\sqrt{2}}\left\langle\{P,\varepsilon_{\mathrm{P}}\}\,\{u_\mu,\nabla^\mu\Delta\}\right\rangle + \lambda_2^{\mathrm{PP}}\left\langle P\,\nabla_\mu\Delta\,P\,\nabla^\mu\Delta\right\rangle + \frac{\lambda_2^{\mathrm{PP}}}{2}\left\langle\varepsilon_{\mathrm{P}}u_\mu\varepsilon_{\mathrm{P}}u^\mu\right\rangle - \\
& -\sqrt{2}\lambda_2^{\mathrm{PP}}\left\langle\varepsilon_{\mathrm{P}}\big(\nabla_\mu\Delta\,P\,u^\mu + u_\mu P\,\nabla^\mu\Delta\big)\right\rangle + \frac{\lambda_2^{\mathrm{PP}}}{4}\left\langle\big[[\Delta,u_\mu],\Delta\big]P\,u^\mu P\right\rangle -
\end{aligned}$$

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$$
-\frac{\lambda_3^{\mathrm{PP}}}{8}\left\langle\{PP,\Delta\}\{\chi_+,\Delta\}\right\rangle-\frac{i\,\lambda_3^{\mathrm{PP}}}{2\sqrt{2}}\left\langle\{P,\varepsilon_{\mathrm{P}}\}\{\chi_-,\Delta\}\right\rangle+\frac{\lambda_3^{\mathrm{PP}}}{2}\left\langle\varepsilon_{\mathrm{P}}^2\,\chi_+\right\rangle, \quad \text{(D.5)}
$$

$$
\begin{aligned}
\Delta\mathcal{L}_2(\mathrm{S},\mathrm{P}) = {} & \frac{\lambda_1^{\mathrm{SP}}}{8}\left\langle\{\nabla_\mu S,P\}\big[[\Delta,u^\mu],\Delta\big]\right\rangle-\frac{\lambda_1^{\mathrm{SP}}}{\sqrt{2}}\left\langle\nabla^\mu\Delta\big(\{\nabla_\mu\varepsilon_{\mathrm{S}},P\}+\{\nabla_\mu S,\varepsilon_{\mathrm{P}}\}\big)\right\rangle+\\
&+\frac{\lambda_1^{\mathrm{SP}}}{4\sqrt{2}}\Big\langle\big[[u_\mu,\Delta],S\big]\big(\{\varepsilon_{\mathrm{P}},u^\mu\}-\sqrt{2}\{P,\nabla^\mu\Delta\}\big)\Big\rangle+\frac{\lambda_1^{\mathrm{SP}}}{2}\left\langle\{\nabla_\mu\varepsilon_{\mathrm{S}},\varepsilon_{\mathrm{P}}\}u^\mu\right\rangle+\\
&+\frac{\lambda_1^{\mathrm{SP}}}{4\sqrt{2}}\left\langle\big[[u_\mu,\Delta],\varepsilon_{\mathrm{S}}\big]\{P,u^\mu\}\right\rangle+\frac{\lambda_1^{\mathrm{SP}}}{8}\left\langle\big[[\Delta,\nabla_\mu\Delta],S\big]\{P,u^\mu\}\right\rangle-\\
&-\frac{i\,\lambda_2^{\mathrm{SP}}}{8}\left\langle\{S,P\}\big\{\Delta,\{\chi_-,\Delta\}\big\}\right\rangle+\frac{\lambda_2^{\mathrm{SP}}}{2\sqrt{2}}\Big\langle\{\Delta,\chi_+\}\Big(\{\varepsilon_{\mathrm{S}},P\}+\{S,\varepsilon_{\mathrm{P}}\}\Big)\Big\rangle+\\
&+\frac{i\,\lambda_2^{\mathrm{SP}}}{2}\left\langle\chi_-\{\varepsilon_{\mathrm{S}},\varepsilon_{\mathrm{P}}\}\right\rangle. \qquad \text{(D.6)}
\end{aligned}
$$

The evaluation of the path integral requires a gaussian rearrangement of the integration variables. However the second-order fluctuation $\Delta\mathcal{L}_{R\chi T}$ does not have this structure due to the terms $\langle PP\,\nabla_\mu\Delta\nabla^\mu\Delta\rangle$, $\langle P\,\nabla_\mu\Delta\,P\,\nabla^\mu\Delta\rangle$, $\langle S\nabla_\mu\Delta\nabla^\mu\Delta\rangle$, $\langle SS\,\nabla_\mu\Delta\nabla^\mu\Delta\rangle$, $\langle S\,\nabla_\mu\Delta\,S\,\nabla^\mu\Delta\rangle$ and $\langle\{\nabla_\mu\varepsilon_{\mathrm{S}},P\}\nabla^\mu\Delta\rangle$ in eqs. (D.4), (D.5), (D.6). A way out is provided by a redefinition of the fields that eliminates the unwanted terms:

$$
\begin{aligned}
\Delta &\to \Delta-\frac{c_d}{F^2}\{\Delta,S\}-\frac{\tilde\lambda_1^{\mathrm{SS}}}{F^2}\{\Delta,SS\}-\frac{2\tilde\lambda_2^{\mathrm{SS}}}{F^2}S\,\Delta\,S-\frac{\tilde\lambda_1^{\mathrm{PP}}}{F^2}\{\Delta,PP\}-\frac{2\tilde\lambda_2^{\mathrm{PP}}}{F^2}P\,\Delta\,P\,,\\
\varepsilon_{\mathrm{S}} &\to \varepsilon_{\mathrm{S}}+\sqrt{2}\lambda_1^{\mathrm{SP}}\{P,\Delta\}-\frac{\sqrt{2}\lambda_1^{\mathrm{SP}}c_d}{F^2}\{P,\{\Delta,S\}\}\,, \qquad \text{(D.7)}
\end{aligned}
$$

where the following constants have been defined:

$$
\begin{aligned}
\tilde\lambda_1^{\mathrm{SS}} &\equiv \lambda_1^{\mathrm{SS}}-\frac{3}{2}\frac{c_d^2}{F^2}\,, \qquad \tilde\lambda_2^{\mathrm{SS}}\equiv\lambda_2^{\mathrm{SS}}-\frac{3}{2}\frac{c_d^2}{F^2}\,,\\
\tilde\lambda_1^{\mathrm{PP}} &\equiv \lambda_1^{\mathrm{PP}}-\left(\lambda_1^{\mathrm{SP}}\right)^2, \qquad \tilde\lambda_2^{\mathrm{PP}}\equiv\lambda_2^{\mathrm{PP}}-\left(\lambda_1^{\mathrm{SP}}\right)^2. \qquad \text{(D.8)}
\end{aligned}
$$

The transformation of the integration measure only yields $\delta^4(0)$ terms which have no effect on the theory [35].[10]

Performing the transformations given by eq. (D.7) on $\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ and keeping only terms with up to two resonances we finally obtain:

$$
\begin{aligned}
\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = {} & -\frac{1}{2}\,\Delta_i\left(d'_\mu d'^\mu+\sigma\right)_{ij}\Delta_j-\frac{1}{2}\,\varepsilon_{\mathrm{S}i}\left(d^\mu d_\mu+k^{\mathrm{S}}\right)_{ij}\varepsilon_{\mathrm{S}j}-\\
&-\frac{1}{2}\,\varepsilon_{\mathrm{P}i}\left(d^\mu d_\mu+k^{\mathrm{P}}\right)_{ij}\varepsilon_{\mathrm{P}j}+\varepsilon_{\mathrm{S}i}\,{a^{\mathrm{S}}}_{ij}\,\Delta_j+\varepsilon_{\mathrm{P}i}\,{a^{\mathrm{P}}}_{ij}\,\Delta_j+\varepsilon_{\mathrm{P}i}\,{a^{\mathrm{SP}}}_{ij}\,\varepsilon_{\mathrm{S}j}+\\
&+\varepsilon_{\mathrm{S}k}\,b^{\mathrm{S}}_{\mu\,ki}\,d^\mu_{ij}\Delta_j+\varepsilon_{\mathrm{P}k}\,b^{\mathrm{P}}_{\mu\,ki}\,d^\mu_{ij}\Delta_j+\varepsilon_{\mathrm{P}k}\,b^{\mathrm{SP}}_{\mu\,ki}\,d^\mu_{ij}\varepsilon_{\mathrm{S}j}\,, \qquad \text{(D.9)}
\end{aligned}
$$

that has the proper gaussian structure and where the following definitions have been introduced:

$$
d^\mu_{ij} = \delta_{ij}\,\partial^\mu+\gamma^\mu_{ij}\big|_\chi\,, \qquad \text{(D.10)}
$$

[10] In dimensional regularization the later result is immediate, as $\delta^d(0)=0$.

$$
\begin{aligned}
d'^{\mu}_{ij} &= d^{\mu}_{ij} + \gamma^{\mu}_{ij}\big|_{\mathrm{R}}\,, && \text{(D.11)}\\
\gamma^{\mu}_{ij}\big|_{\chi} &= -\frac{1}{2}\,\langle \Gamma^{\mu}[\lambda_i,\lambda_j]\rangle\,, && \text{(D.12)}\\
\gamma^{\mu}_{ij}\big|_{\mathrm{R}} &= \frac{c_d\lambda_1^{\mathrm{SP}}}{2F^2}\,\langle\{P,\lambda_i\}\{u^{\mu},\lambda_j\}\rangle + \Big(-\frac{1}{16F^2}+\frac{c_d^2}{8F^4}\Big)\,\langle[S,\nabla^{\mu}S][\lambda_i,\lambda_j]\rangle - \\
&\quad -\frac{1}{16F^2}\,\langle[P,\nabla^{\mu}P][\lambda_i,\lambda_j]\rangle - \frac{\lambda_1^{\mathrm{SP}}}{16F^2}\,\big\langle\big[S,\{P,u^{\mu}\}\big][\lambda_i,\lambda_j]\big\rangle + \\
&\quad +\frac{\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}}{F^2}\,\big\langle\{P,\lambda_i\}\big(\lambda_j S\,u^{\mu} + u^{\mu}S\,\lambda_j\big)\big\rangle + \frac{\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}}{2F^2}\,\big\langle\big\{S,\{P,\lambda_i\}\big\}\{u^{\mu},\lambda_j\}\big\rangle - \\
&\quad -\frac{c_d^2\lambda_1^{\mathrm{SP}}}{2F^4}\,\big\langle\{S,\lambda_i\}\big[[P,u^{\mu}],\lambda_j\big]\big\rangle - \Big\{i\leftrightarrow j\Big\}\,, && \text{(D.13)}\\
k^{\mathrm{S}}{}_{ij} &= \delta_{ij}M_S^2 - \frac{\lambda_1^{\mathrm{SS}}}{2}\,\langle u^{\mu}u_{\mu}\{\lambda_i,\lambda_j\}\rangle - \lambda_2^{\mathrm{SS}}\,\langle\lambda_i u_{\mu}\lambda_j u^{\mu}\rangle - \frac{\lambda_3^{\mathrm{SS}}}{2}\,\langle\chi_+\{\lambda_i,\lambda_j\}\rangle\ , && \text{(D.14)}\\
k^{\mathrm{P}}{}_{ij} &= \delta_{ij}M_P^2 - \frac{\lambda_1^{\mathrm{PP}}}{2}\,\langle u^{\mu}u_{\mu}\{\lambda_i,\lambda_j\}\rangle - \lambda_2^{\mathrm{PP}}\,\langle\lambda_i u_{\mu}\lambda_j u^{\mu}\rangle - \frac{\lambda_3^{\mathrm{PP}}}{2}\,\langle\chi_+\{\lambda_i,\lambda_j\}\rangle\,, && \text{(D.15)}\\
\sigma_{ij} &= \frac{1}{16}\,\Big\langle\chi_+\{\hat\lambda_i,\hat\lambda_j\}\Big\rangle - \frac{1}{16}\,\Big\langle[u_{\mu},\hat\lambda_i][u^{\mu},\hat\lambda_j]\Big\rangle - \\
&\quad -\frac{c_d}{4F^2}\,\langle\nabla^2 S\{\lambda_i,\lambda_j\}\rangle + \frac{c_m}{8F^2}\,\Big\langle\{S,\hat\lambda_i\}\{\chi_+,\hat\lambda_j\}\Big\rangle + \frac{c_d}{8F^2}\,\Big\langle\{S,u^{\mu}\}\big[[u_{\mu},\hat\lambda_i],\hat\lambda_j\big]\Big\rangle - \\
&\quad -\frac{c_d\lambda_1^{\mathrm{SP}}}{2F^2}\bigg(\Big\langle\{\nabla_{\mu}P,\hat\lambda_i\}\{u^{\mu},\hat\lambda_j\}\Big\rangle + \Big\langle\{P,\hat\lambda_i\}\{\nabla_{\mu}u^{\mu},\hat\lambda_j\}\Big\rangle\bigg) + \\
&\quad +\frac{i}{2F^2}\bigg(\frac{d_m}{4} + c_m\lambda_1^{\mathrm{SP}}\bigg)\Big\langle\{P,\hat\lambda_i\}\{\chi_-,\hat\lambda_j\}\Big\rangle - \frac{1}{32F^2}\,\big\langle\big[S,[u^{\mu},\lambda_i]\big]\big[S,[u_{\mu},\lambda_j]\big]\big\rangle + \\
&\quad +\frac{1}{8F^2}\,\Big\langle[u^{\mu},\lambda_i]\big[u_{\mu},\tilde\lambda_1^{\mathrm{SS}}\{SS,\lambda_j\} + 2\tilde\lambda_2^{\mathrm{SS}}S\lambda_j S)\big]\Big\rangle + \\
&\quad +\frac{1}{8F^2}\,\Big\langle[u^{\mu},\lambda_i]\big[\lambda_j,\big(\lambda_1^{\mathrm{SS}}\{SS,u_{\mu}\} + 2\lambda_2^{\mathrm{SS}}Su_{\mu}S)\big]\big)\Big\rangle - \\
&\quad -\frac{1}{8F^2}\,\Big\langle\{\chi_+,\lambda_i\}\Big(\big(\tilde\lambda_1^{\mathrm{SS}} - \lambda_3^{\mathrm{SS}}\big)\{SS,\lambda_j\} + 2\tilde\lambda_2^{\mathrm{SS}}S\lambda_j S\Big)\Big\rangle - \\
&\quad -\frac{c_d^2}{2F^4}\,\langle\{\nabla^{\mu}S,\lambda_i\}\{\nabla_{\mu}S,\lambda_j\}\rangle - \frac{c_d^2}{4F^4}\,\langle\{S,\lambda_i\}\{\nabla^2 S,\lambda_j\}\rangle - \\
&\quad -\frac{\tilde\lambda_1^{\mathrm{SS}}}{4F^2}\,\langle\nabla^2 S^2\{\lambda_i,\lambda_j\}\rangle - \frac{\tilde\lambda_2^{\mathrm{SS}}}{F^2}\,\big\langle\lambda_i\nabla_{\mu}\big(\nabla^{\mu}S\lambda_j S\big)\big\rangle + \\
&\quad +\frac{M_S^2\big(\lambda_1^{\mathrm{SP}}\big)^2}{2F^2}\,\langle\{P,\lambda_i\}\{P,\lambda_j\}\rangle - \frac{\tilde\lambda_1^{\mathrm{PP}}}{8F^2}\,\langle\{\chi_+,\lambda_i\}\{PP,\lambda_j\}\rangle - \\
&\quad -\frac{\tilde\lambda_2^{\mathrm{PP}}}{4F^2}\,\langle\{\chi_+,\lambda_i\}P\,\lambda_j P\rangle - \frac{\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}}}{2F^2}\,\big\langle\big\{P,\{P,\lambda_i\}\big\}\{\chi_+,\lambda_j\}\big\rangle + \\
&\quad +\frac{\lambda_3^{\mathrm{PP}}}{8F^2}\,\langle\{PP,\lambda_i\}\{\chi_+,\lambda_j\}\rangle - \frac{\lambda_3^{\mathrm{SS}}\big(\lambda_1^{\mathrm{SP}}\big)^2}{F^2}\,\langle\{P,\lambda_i\}\{P,\lambda_j\}\chi_+\rangle - \\
&\quad -\frac{1}{32F^2}\,\big\langle\big[P,[u^{\mu},\lambda_i]\big]\big[P,[u_{\mu},\lambda_j]\big]\big\rangle + \frac{c_d^2\big(\lambda_1^{\mathrm{SP}}\big)^2}{4F^4}\,\big\langle\big[[u^{\mu},P],\lambda_i\big]\big[[u_{\mu},P],\lambda_j\big]\big\rangle - \\
&\quad -\frac{\big(\lambda_1^{\mathrm{SP}}\big)^2}{4F^2}\,\big\langle\big[[u^{\mu},\lambda_i],\{P,\lambda_j\}\big]\{P,u_{\mu}\}\big\rangle - \\
&\quad -\frac{1}{8F^2}\,\Big\langle\Big(\lambda_1^{\mathrm{PP}}\{u_{\mu},PP\} + 2\lambda_2^{\mathrm{PP}}P\,u_{\mu}P\Big)\big[[\lambda_i,u^{\mu}],\lambda_j\big]\Big\rangle -
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{(\lambda_1^{\rm SP})^2}{F^2}\left\langle u^\mu\{P,\lambda_i\}\Big(\lambda_1^{\rm SS}\{P,\lambda_j\}u_\mu+\lambda_2^{\rm SS}u_\mu\{P,\lambda_j\}\Big)\right\rangle+\\
&+\frac{\tilde\lambda_1^{\rm PP}}{8F^2}\left\langle\big[u_\mu,\{PP,\lambda_i\}\big][u^\mu,\lambda_j]\right\rangle+\frac{\tilde\lambda_2^{\rm PP}}{4F^2}\left\langle[u_\mu,P\,\lambda_iP][u^\mu,\lambda_j]\right\rangle-\\
&-\frac{\tilde\lambda_1^{\rm PP}}{4F^2}\left\langle\nabla^2P^2\{\lambda_i,\lambda_j\}\right\rangle-\frac{\tilde\lambda_2^{\rm PP}}{F^2}\left\langle\lambda_i\nabla_\mu\big(\nabla^\mu P\lambda_jP\big)\right\rangle-\\
&-\frac{(\lambda_1^{\rm SP})^2}{2F^2}\left\langle\{\nabla_\mu P,\lambda_i\}\{\nabla^\mu P,\lambda_j\}\right\rangle+\frac{i\lambda_2^{\rm SP}}{8F^2}\left\langle\{S,P\}\big\{\lambda_i,\{\chi_-,\lambda_j\}\big\}\right\rangle+\\
&+\frac{i\,\lambda_3^{\rm SS}\lambda_1^{\rm SP}}{2F^2}\left\langle\big\{S,\{P,\lambda_i\}\big\}\{\chi_-,\lambda_j\}\right\rangle-\frac{c_d^2\lambda_1^{\rm SP}}{2F^4}\left\langle\{P,\lambda_i\}\big\{u^\mu,\{\nabla_\mu S,\lambda_j\}\big\}\right\rangle-\\
&-\frac{\lambda_2^{\rm SS}\lambda_1^{\rm SP}}{F^2}\left\langle\nabla_\mu\Big(\{P,\lambda_i\}\big(\lambda_jSu^\mu+u^\mu S\lambda_j\big)\Big)\right\rangle-\frac{\lambda_1^{\rm SS}\lambda_1^{\rm SP}}{2F^2}\left\langle\{P,\lambda_i\}\nabla_\mu\big\{S,\{u^\mu,\lambda_j\}\big\}\right\rangle+\\
&+\frac{\lambda_1^{\rm SP}}{8F^2}\left\langle[u^\mu,\lambda_i]\Big(\big[\lambda_j,\{\nabla_\mu S,P\}\big]+2\big[\nabla^\mu S,\{P,\lambda_j\}\big]-2\big[S,\{\nabla^\mu P,\lambda_j\}\big]\Big)\right\rangle+\\
&+\frac{\lambda_1^{\rm SP}}{2F^2}\left\langle\{u^\mu,\lambda_i\}\Big(\frac{c_d^2}{F^2}\big\{P,\{\nabla_\mu S,\lambda_j\}\big\}-\lambda_1^{\rm SS}\big\{S,\{\nabla_\mu P,\lambda_j\}\big\}\Big)\right\rangle+\Big\{i\leftrightarrow j\Big\}, \qquad \text{(D.16)}
\end{aligned}
$$

$$
\begin{aligned}
{a^{\rm S}}_{ij}=&-\frac{i\,c_m}{2\sqrt2F}\left\langle\chi_-\{\lambda_i,\hat\lambda_j\}\right\rangle-\frac{1}{2\sqrt2F}\left\langle[\nabla^\mu S,\lambda_i][u_\mu,\hat\lambda_j]\right\rangle-\\
&-\frac{1}{4\sqrt2F}\left\langle[S,\lambda_i][\nabla^\mu u_\mu,\hat\lambda_j]\right\rangle+\frac{c_d^2}{\sqrt2F^3}\left\langle\{u^\mu,\lambda_i\}\{\nabla_\mu S,\lambda_j\}\right\rangle-\\
&-\frac{i\,\lambda_3^{\rm SS}}{2\sqrt2F}\left\langle\{S,\lambda_i\}\{\chi_-,\hat\lambda_j\}\right\rangle-\frac{M_S^2\lambda_1^{\rm SP}}{\sqrt2F}\left\langle P\{\lambda_i,\hat\lambda_j\}\right\rangle+\\
&+\frac{\lambda_2^{\rm SP}}{2\sqrt2F}\left\langle\{P,\lambda_i\}\{\chi_+,\hat\lambda_j\}\right\rangle+\frac{\lambda_1^{\rm SP}\lambda_3^{\rm SS}}{\sqrt2F}\left\langle\{\chi_+,\lambda_i\}\{P,\hat\lambda_j\}\right\rangle-\\
&-\frac{\lambda_1^{\rm SP}}{\sqrt2F}\left\langle\nabla^2P\{\lambda_i,\hat\lambda_j\}\right\rangle-\frac{\lambda_1^{\rm SP}}{4\sqrt2F}\left\langle\{P,u^\mu\}\big[\lambda_i,[u_\mu,\hat\lambda_j]\big]\right\rangle+\\
&+\frac{\lambda_1^{\rm SP}}{\sqrt2F}\left\langle\{P,\hat\lambda_j\}\Big(\lambda_1^{\rm SS}\{u_\mu u^\mu,\lambda_i\}+2\lambda_2^{\rm SS}u_\mu\lambda_iu^\mu\Big)\right\rangle+\\
&+\frac{c_d}{\sqrt2F^3}\left\langle\{u^\mu,\lambda_i\}\,\Big(\tilde\lambda_1^{\rm SS}\{\nabla_\mu(SS),\lambda_j\}+2\tilde\lambda_2^{\rm SS}\nabla_\mu(S\lambda_jS)\Big)\right\rangle+\\
&+\frac{\sqrt2\lambda_2^{\rm SS}\,c_d}{F^3}\left\langle\{\nabla^\mu S,\lambda_j\}\Big(Su_\mu\lambda_i+\lambda_iu_\mu S\Big)\right\rangle+\frac{\lambda_1^{\rm SS}\,c_d}{\sqrt2F^3}\left\langle\{S,\lambda_i\}\big\{u_\mu,\{\nabla^\mu S,\lambda_j\}\big\}\right\rangle+\\
&+\frac{c_d}{4\sqrt2F^3}\left\langle[S,\lambda_i]\big[u_\mu,\{\nabla^\mu S,\lambda_j\}\big]\right\rangle+\\
&+\frac{i\,c_m}{2\sqrt2F^3}\left\langle\{\chi_-,\lambda_i\}\Big(\tilde\lambda_1^{\rm SS}\{SS,\lambda_j\}+2\tilde\lambda_2^{\rm SS}S\lambda_jS\Big)\right\rangle+\\
&+\frac{i\,c_m}{2\sqrt2F^3}\left\langle\{\chi_-,\lambda_i\}\Big(\tilde\lambda_1^{\rm PP}\{PP,\lambda_j\}+2\tilde\lambda_2^{\rm PP}P\,\lambda_jP\Big)\right\rangle+\\
&+\frac{c_d}{\sqrt2F^3}\left\langle\{u^\mu,\lambda_i\}\Big(\tilde\lambda_1^{\rm PP}\{\nabla_\mu(PP),\lambda_j\}+2\tilde\lambda_2^{\rm PP}\nabla_\mu(P\,\lambda_jP)\Big)\right\rangle+\\
&+\frac{c_d\lambda_1^{\rm SP}}{\sqrt2F^3}\left\langle\{\nabla_\mu P,\lambda_i\}\{\nabla^\mu S,\lambda_j\}\right\rangle, \qquad \text{(D.17)}
\end{aligned}
$$

$$
\begin{aligned}
a^{\mathrm{P}}{}_{ij} &= \frac{d_m}{2\sqrt{2}F}\left\langle \chi_+\{\lambda_i,\hat{\lambda}_j\}\right\rangle + \frac{\lambda_2^{\mathrm{SP}}}{2\sqrt{2}F}\left\langle \{S,\lambda_i\}\{\chi_+,\hat{\lambda}_j\}\right\rangle - \\
&\quad -\frac{\lambda_1^{\mathrm{SP}}}{4\sqrt{2}F}\left\langle \{u_\mu,\lambda_i\}\big[S,[u^\mu,\hat{\lambda}_j]\big]\right\rangle - \\
&\quad -\frac{1}{4\sqrt{2}F}\left\langle \Big([P,\lambda_i][\nabla^\mu u_\mu,\hat{\lambda}_j]+2[\nabla^\mu P,\lambda_i][u_\mu,\hat{\lambda}_j]\Big)\right\rangle + \\
&\quad +\frac{i\,\lambda_1^{\mathrm{SP}}}{\sqrt{2}F}\lambda_2^{\mathrm{SP}}\left\langle \{\chi_-,\lambda_i\}\{P,\hat{\lambda}_j\}\right\rangle - \frac{i\,\lambda_3^{\mathrm{PP}}}{2\sqrt{2}F}\left\langle \{P,\lambda_i\}\{\chi_-,\hat{\lambda}_j\}\right\rangle + \\
&\quad +\frac{\left(\lambda_1^{\mathrm{SP}}\right)^2}{\sqrt{2}F}\left\langle \{u^\mu,\lambda_i\}\{\nabla_\mu P,\hat{\lambda}_j\}\right\rangle + \frac{c_d\lambda_1^{\mathrm{SP}}}{\sqrt{2}F^3}\left\langle \{\nabla_\mu S,\lambda_i\}\{\nabla^\mu S,\lambda_j\}\right\rangle - \\
&\quad -\frac{d_m}{2\sqrt{2}F^3}\left\langle \{\chi_+,\lambda_i\}\Big(\tilde{\lambda}_1^{\mathrm{SS}}\{SS,\lambda_j\}+2\tilde{\lambda}_2^{\mathrm{SS}}S\,\lambda_j S+\tilde{\lambda}_1^{\mathrm{PP}}\{PP,\lambda_j\}+2\tilde{\lambda}_2^{\mathrm{PP}}P\,\lambda_j P\Big)\right\rangle - \\
&\quad -\frac{c_d\left(\lambda_1^{\mathrm{SP}}\right)^2}{\sqrt{2}F^3}\left\langle \{u^\mu,\lambda_i\}\big\{P,\{\nabla_\mu S,\lambda_j\}\big\}\right\rangle + \frac{c_d}{4\sqrt{2}F^3}\left\langle [P,\lambda_i]\big[u_\mu,\{\nabla^\mu S,\lambda_j\}\big]\right\rangle + \\
&\quad +\frac{c_d}{\sqrt{2}F^3}\left\langle \{\nabla^\mu S,\lambda_j\}\Big(\lambda_1^{\mathrm{PP}}\big\{u_\mu,\{P,\lambda_i\}\big\}+2\lambda_2^{\mathrm{PP}}\left(P\,u_\mu\lambda_i+\lambda_i u_\mu P\right)\Big)\right\rangle\,, \qquad \text{(D.18)}\\
a^{\mathrm{SP}}{}_{ij} &= \frac{i\,\lambda_2^{\mathrm{SP}}}{2}\left\langle \chi_-\{\lambda_i,\lambda_j\}\right\rangle\,, \qquad \text{(D.19)}\\
b^{\mathrm{S}\mu}_{\ \ ij} &= -\frac{c_d}{\sqrt{2}F}\left\langle u^\mu\{\lambda_i,\hat{\lambda}_j\}\right\rangle - \frac{1}{4\sqrt{2}F}\left\langle [S,\lambda_i][u^\mu,\hat{\lambda}_j]\right\rangle - \frac{\lambda_1^{\mathrm{SS}}}{\sqrt{2}F}\left\langle \{S,\lambda_i\}\{u^\mu,\hat{\lambda}_j\}\right\rangle - \\
&\quad -\frac{\sqrt{2}\lambda_2^{\mathrm{SS}}}{F}\left\langle S\Big(u^\mu\lambda_i\hat{\lambda}_j+\hat{\lambda}_j\lambda_i u^\mu\Big)\right\rangle - \frac{\lambda_1^{\mathrm{SP}}}{\sqrt{2}F}\left\langle \nabla^\mu P\{\lambda_i,\hat{\lambda}_j\}\right\rangle + \qquad \text{(D.20)}\\
&\quad +\frac{c_d}{\sqrt{2}F^3}\left\langle \{u^\mu,\lambda_i\}\Big(\tilde{\lambda}_1^{\mathrm{SS}}\{SS,\lambda_j\}+2\tilde{\lambda}_2^{\mathrm{SS}}S\lambda_j S+\tilde{\lambda}_1^{\mathrm{PP}}\{PP,\lambda_j\}+2\tilde{\lambda}_2^{\mathrm{PP}}P\lambda_j P\Big)\right\rangle,\\
b^{\mathrm{P}\mu}_{\ \ ij} &= -\frac{\lambda_1^{\mathrm{SP}}}{\sqrt{2}F}\left\langle \nabla^\mu S\{\lambda_i,\hat{\lambda}_j\}\right\rangle - \frac{1}{4\sqrt{2}F}\left\langle [P,\lambda_i][u^\mu,\hat{\lambda}_j]\right\rangle - \frac{\lambda_1^{\mathrm{PP}}}{\sqrt{2}F}\left\langle \{P,\lambda_i\}\{u^\mu,\hat{\lambda}_j\}\right\rangle - \\
&\quad -\frac{\sqrt{2}\lambda_2^{\mathrm{PP}}}{F}\left\langle P\Big(u^\mu\lambda_i\hat{\lambda}_j+\hat{\lambda}_j\lambda_i u^\mu\Big)\right\rangle + \frac{\left(\lambda_1^{\mathrm{SP}}\right)^2}{\sqrt{2}F}\left\langle \{u^\mu,\lambda_i\}\{P,\hat{\lambda}_j\}\right\rangle, \qquad \text{(D.21)}\\
b^{\mathrm{SP}\mu}_{\ \ \ ij} &= \frac{\lambda_1^{\mathrm{SP}}}{2}\left\langle u^\mu\{\lambda_i,\lambda_j\}\right\rangle, \qquad \text{(D.22)}
\end{aligned}
$$

and the following definitions have been used,

$$
\hat{\lambda}_i \equiv \lambda_i - \frac{c_d}{F^2}\{\lambda_i,S\}\,, \qquad \nabla_\mu\left(A\,\lambda_i\,B\right) \equiv \nabla_\mu A\,\lambda_i\,B + A\,\lambda_i\,\nabla_\mu B\,, \tag{D.23}
$$

where A and B are any chiral tensor or resonance field.

As commented in the text we can write eq. (D.9) as:

$$
\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = -\frac{1}{2}\,\eta\left(\Sigma_\mu\,\Sigma^\mu+\Lambda\right)\eta^\top\,, \tag{D.24}
$$

where η collects the fluctuation fields, $\eta = \Big(\Delta_i,\varepsilon_{\mathrm{S}j},\varepsilon_{\mathrm{P}k}\Big)$, $i,j,k = 0,\ldots,8$, $\eta^\top$ is its transposed and Σ_μ and Λ are defined as:

$$
\left(\Sigma_\mu\right)_{ij} = \delta_{ij}\,\partial_\mu + \left(Y_\mu\right)_{ij}\,, \tag{D.25}
$$

$$
(Y_\mu)_{ij} = \begin{pmatrix} \gamma'_\mu & \frac{1}{2} b_\mu^{\mathrm{S}\,\top} & \frac{1}{2} b_\mu^{\mathrm{P}\,\top} \\ -\frac{1}{2} b_\mu^{\mathrm{S}} & \gamma_\mu & \frac{1}{2} b_\mu^{\mathrm{SP}\,\top} \\ -\frac{1}{2} b_\mu^{\mathrm{P}} & -\frac{1}{2} b_\mu^{\mathrm{SP}} & \gamma_\mu \end{pmatrix}_{ij} , \tag{D.26}
$$

$$
(\Lambda)_{ij} = \begin{pmatrix} \begin{matrix} \sigma + \frac{1}{4} b_\mu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\mu} \\ + \frac{1}{4} b_\mu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\mu} \end{matrix} & \begin{matrix} -a^{\mathrm{S}\,\top} + \frac{1}{2} \tilde{d}_-^{\mu} b_\mu^{\mathrm{S}\,\top} \\ + \frac{1}{4} b_\mu^{\mathrm{P}\,\top} b^{\mathrm{SP}\,\mu} \end{matrix} & \begin{matrix} -a^{\mathrm{P}\,\top} + \frac{1}{2} \tilde{d}_-^{\mu} b_\mu^{\mathrm{P}\,\top} \\ - \frac{1}{4} b_\mu^{\mathrm{S}\,\top} b^{\mathrm{SP}\,\mu\,\top} \end{matrix} \\ \\ \begin{matrix} -a^{\mathrm{S}} + \frac{1}{2} \bar{d}_+^{\mu} b_\mu^{\mathrm{S}} \\ + \frac{1}{4} b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{P}\,\mu} \end{matrix} & \begin{matrix} k^{\mathrm{S}} + \frac{1}{4} b^{\mathrm{S}\,\mu} b_\mu^{\mathrm{S}\,\top} \\ + \frac{1}{4} b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\mu} \end{matrix} & \begin{matrix} -a^{\mathrm{SP}\,\top} + \frac{1}{2} \hat{d}^{\mu} b_\mu^{\mathrm{SP}\,\top} \\ + \frac{1}{4} b^{\mathrm{S}\,\mu} b_\mu^{\mathrm{P}\,\top} \end{matrix} \\ \\ \begin{matrix} -a^{\mathrm{P}} + \frac{1}{2} \bar{d}_+^{\mu} b_\mu^{\mathrm{P}} \\ - \frac{1}{4} b^{\mathrm{SP}\,\mu} b_\mu^{\mathrm{S}} \end{matrix} & \begin{matrix} -a^{\mathrm{SP}} + \frac{1}{2} \hat{d}^{\mu} b_\mu^{\mathrm{SP}} \\ + \frac{1}{4} b^{\mathrm{P}\,\mu} b_\mu^{\mathrm{S}\,\top} \end{matrix} & \begin{matrix} k^{\mathrm{P}} + \frac{1}{4} b^{\mathrm{P}\,\mu} b_\mu^{\mathrm{P}\,\top} \\ + \frac{1}{4} b^{\mathrm{SP}\,\mu} b_\mu^{\mathrm{SP}\,\top} \end{matrix} \end{pmatrix}_{ij} . \tag{D.27}
$$

Here some new expressions have been defined:

$$
\begin{aligned}
\gamma^\mu &= \gamma^\mu\big|_\chi , \\
\gamma'^\mu &= \gamma^\mu\big|_\chi + \gamma^\mu\big|_{\mathrm{R}} , \\
\hat{d}^\mu X &= \partial^\mu X + [\gamma^\mu , X] , \\
\tilde{d}_\pm^\mu X &= \hat{d}^\mu X \pm \left(\gamma'^\mu - \gamma^\mu\right) X , \\
\bar{d}_\pm^\mu X &= \hat{d}^\mu X \pm X \left(\gamma'^\mu - \gamma^\mu\right) .
\end{aligned} \tag{D.28}
$$

E. Results: operators and coefficients of the β-function

In this appendix we collect part of our results. In table 1 we have written the operators that involve only Goldstone fields and/or external currents. The coefficients of the β-function are generated by full $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$, i.e. including both scalar and pseudoscalar resonances. They can be compared with the study of one-loop renormalization of $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ χPT up to $\mathcal{O}(p^4)$ in ref. [30]. The operators in this table constitute a minimal basis. If one wishes to disentangle the pseudoscalar contributions one has to cancel the corresponding resonance couplings and substitute $N/16 \to N/24$ in γ_4, $0 \to N/48$ in γ_7, $N/4 \to N/6$ in γ_{13}, $-N/4 \to -N/6$ in γ_{14} and $-N/8 \to -N/12$ in γ_{15}.

In tables 2, 3 we collect the operators involving one and two scalar resonances, respectively. The β-function coefficients are those obtained by considering Goldstone and scalar resonances only. The full result when pseudoscalar resonances are also introduced in the theory can be looked up at `http://ific.uv.es/quiral/rt1loop.html` or upon

i	$\mathcal{O}_i$	γ_i
1	$\langle u\cdot u\rangle$	$-2N\lambda_1^{\rm PP}M_P^2+1/2NM_P^2(\lambda_1^{\rm SP})^2-2N\lambda_1^{\rm SS}M_S^2+1/2NM_S^2(\lambda_1^{\rm SP})^2+NF^{-2}c_d^2M_S^2$
2	$\langle\chi_+\rangle$	$-2N\lambda_3^{\rm PP}M_P^2-2N\lambda_3^{\rm SS}M_S^2$
3	$-\langle u_\mu\rangle^2$	$2\lambda_2^{\rm PP}M_P^2-1/2M_P^2(\lambda_1^{\rm SP})^2+2\lambda_2^{\rm SS}M_S^2-1/2M_S^2(\lambda_1^{\rm SP})^2-c_d^2F^{-2}M_S^2$
4	$\langle u_\mu u_\nu u^\mu u^\nu\rangle$	$1/6NF^{-4}c_d^4-1/12N(\lambda_1^{\rm SP})^2+1/24N(\lambda_1^{\rm SP})^4+1/16N+1/6NF^{-2}c_d^2(\lambda_1^{\rm SP})^2-1/6NF^{-2}c_d^2$
5	$\langle u\cdot u\rangle^2$	$1/2F^{-4}c_d^4-1/2\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+(\lambda_1^{\rm PP})^2-1/2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+(\lambda_1^{\rm SS})^2+1/8(\lambda_1^{\rm SP})^4+1/16-F^{-2}c_d^2\lambda_1^{\rm SS}+1/2F^{-2}c_d^2(\lambda_1^{\rm SP})^2-1/4F^{-2}c_d^2$
6	$\langle u_\mu u_\nu\rangle^2$	$F^{-4}c_d^4-\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2+2(\lambda_2^{\rm PP})^2-\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2+2(\lambda_2^{\rm SS})^2+1/4(\lambda_1^{\rm SP})^4+1/8-2F^{-2}c_d^2\lambda_2^{\rm SS}+F^{-2}c_d^2(\lambda_1^{\rm SP})^2-1/2F^{-2}c_d^2$
7	$\langle u\cdot uu\cdot u\rangle$	$1/3NF^{-4}c_d^4-1/2N\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+N(\lambda_1^{\rm PP})^2-1/2N\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+N(\lambda_1^{\rm SS})^2+1/12N(\lambda_1^{\rm SP})^2+1/12N(\lambda_1^{\rm SP})^4-NF^{-2}c_d^2\lambda_1^{\rm SS}+1/3NF^{-2}c_d^2(\lambda_1^{\rm SP})^2-1/12NF^{-2}c_d^2$
8	$\langle\chi_+\rangle\langle u\cdot u\rangle$	$F^{-4}c_d^3c_m+2\lambda_1^{\rm PP}\lambda_3^{\rm PP}-1/2\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2+2\lambda_1^{\rm SS}\lambda_3^{\rm SS}-1/2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2+1/8+F^{-2}d_mc_d\lambda_1^{\rm SP}+F^{-2}c_dc_m(\lambda_1^{\rm SP})^2-1/2F^{-2}c_dc_m-F^{-2}c_d^2\lambda_3^{\rm SS}-1/4F^{-2}c_d^2$
9	$\langle\chi_+u\cdot u\rangle$	$NF^{-4}c_d^3c_m+2N\lambda_1^{\rm PP}\lambda_3^{\rm PP}-1/2N\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2+2N\lambda_1^{\rm SS}\lambda_3^{\rm SS}-1/2N\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2+1/8N+NF^{-2}d_mc_d\lambda_1^{\rm SP}+NF^{-2}c_dc_m(\lambda_1^{\rm SP})^2-1/2NF^{-2}c_dc_m-NF^{-2}c_d^2\lambda_3^{\rm SS}-1/4NF^{-2}c_d^2$
10	$\langle\chi_+\rangle^2$	$F^{-4}c_d^2c_m^2+(\lambda_3^{\rm PP})^2+(\lambda_3^{\rm SS})^2+1/16+2F^{-2}d_mc_m\lambda_1^{\rm SP}+F^{-2}d_m^2-1/2F^{-2}c_dc_m+F^{-2}c_m^2(\lambda_1^{\rm SP})^2$
11	$\langle\chi_-\rangle^2$	$\lambda_1^{\rm SP}\lambda_2^{\rm SP}-1/8(\lambda_1^{\rm SP})^2-2(\lambda_2^{\rm SP})^2+F^{-2}d_mc_d\lambda_1^{\rm SP}-2F^{-2}d_mc_m\lambda_1^{\rm SP}-F^{-2}d_m^2(\lambda_1^{\rm SP})^2+F^{-2}c_dc_m-1/4F^{-2}c_d^2-F^{-2}c_m^2$
12	$1/2\left\langle\chi_+^2+\chi_-^2\right\rangle$	$NF^{-4}c_d^2c_m^2+N(\lambda_3^{\rm PP})^2+N(\lambda_3^{\rm SS})^2+1/16N+NF^{-2}d_m^2+1/2NF^{-2}c_dc_m+NF^{-2}c_m^2(\lambda_1^{\rm SP})^2+N\lambda_1^{\rm SP}\lambda_2^{\rm SP}-1/8N(\lambda_1^{\rm SP})^2-2N(\lambda_2^{\rm SP})^2+NF^{-2}d_mc_d\lambda_1^{\rm SP}-NF^{-2}d_m^2(\lambda_1^{\rm SP})^2-1/4NF^{-2}c_d^2-NF^{-2}c_m^2$
13	$-i\left\langle f_+^{\mu\nu}u_\mu u_\nu\right\rangle$	$-1/6N(\lambda_1^{\rm SP})^2+1/4N-1/3NF^{-2}c_d^2$
14	$1/4\left\langle f_{\mu\nu}^{+2}-f_{\mu\nu}^{-2}\right\rangle$	$-1/4N+1/6N(\lambda_1^{\rm SP})^2+1/3NF^{-2}c_d^2$
15	$1/2\left\langle f_{\mu\nu}^{+2}+f_{\mu\nu}^{-2}\right\rangle$	$-1/8N-1/12N(\lambda_1^{\rm SP})^2-1/6NF^{-2}c_d^2$
16	$1/4\left\langle\chi_+^2-\chi_-^2\right\rangle$	$2NF^{-4}c_d^2c_m^2+2N(\lambda_3^{\rm PP})^2+2N(\lambda_3^{\rm SS})^2+1/8N+8F^{-2}Nd_mc_m\lambda_1^{\rm SP}+2NF^{-2}d_m^2-3NF^{-2}c_dc_m+2NF^{-2}c_m^2(\lambda_1^{\rm SP})^2-2N\lambda_1^{\rm SP}\lambda_2^{\rm SP}+1/4N(\lambda_1^{\rm SP})^2+4N(\lambda_2^{\rm SP})^2-2NF^{-2}d_mc_d\lambda_1^{\rm SP}+2NF^{-2}d_m^2(\lambda_1^{\rm SP})^2+1/2NF^{-2}c_d^2+2NF^{-2}c_m^2$
17	$-\langle u_\mu\rangle\langle u^\mu u\cdot u\rangle$	$-4\lambda_1^{\rm PP}\lambda_2^{\rm PP}+\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-4\lambda_1^{\rm SS}\lambda_2^{\rm SS}+\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2-1/2(\lambda_1^{\rm SP})^4+1/4+2F^{-2}c_d^2\lambda_1^{\rm SS}+2F^{-2}c_d^2\lambda_2^{\rm SS}-F^{-2}c_d^2$
18	$\langle u_\mu\rangle\langle u^\mu\chi_+\rangle$	$-2F^{-4}c_d^3c_m+4\lambda_2^{\rm PP}\lambda_3^{\rm PP}-\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2+4\lambda_2^{\rm SS}\lambda_3^{\rm SS}-\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2-1/4-2F^{-2}d_mc_d\lambda_1^{\rm SP}-2F^{-2}c_dc_m(\lambda_1^{\rm SP})^2+F^{-2}c_dc_m-2F^{-2}c_d^2\lambda_3^{\rm SS}+1/2F^{-2}c_d^2$

Table 1: Operators involving only Goldstone bosons and external currents and their β-function coefficients at one loop, when both scalar and pseudoscalar resonances are included.

request from the authors. We would like to emphasize that the basis of operators has been simplified by the use of the EOM in appendix B but possible U(3) algebraic relations have not been employed. Accordingly our basis is not necessarily minimal. In the tables N is the number of flavours and $u\cdot u\equiv u_\mu u^\mu$.

i	$\mathcal{O}_i^S$	γ_i^S
1	$\langle S\, u\cdot u\rangle$	$-3\,N\,F^{-4}\,c_d^3\,M_S^2+2\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,M_S^2+4\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}\,M_S^2$
2	$\langle S\rangle\,\langle u\cdot u\rangle$	$-3\,F^{-4}\,c_d^3\,M_S^2+2\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,M_S^2+F^{-2}\,c_d\,M_S^2$
3	$\langle u^\mu\rangle\,\langle u_\mu\,S\rangle$	$-2\,F^{-4}\,c_d^3\,M_S^2+4\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,M_S^2+4\,F^{-2}\,c_d\,\lambda_2^{\rm SS}\,M_S^2-F^{-2}\,c_d\,M_S^2$
4	$\langle S\,\chi_+\rangle$	$-2\,N\,F^{-4}\,c_d^2\,c_m\,M_S^2+1/2\,N\,F^{-2}\,c_d\,M_S^2$
5	$\langle S\rangle\,\langle\chi_+\rangle$	$-2\,F^{-4}\,c_d^2\,c_m\,M_S^2+1/2\,F^{-2}\,c_d\,M_S^2$
6	$\langle S\,u_\mu u_\nu u^\mu u^\nu\rangle$	$-2/3\,N\,F^{-6}\,c_d^5+2/3\,N\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+4/3\,N\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}+1/6\,N\,F^{-4}\,c_d^3-1/3\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}+1/12\,N\,F^{-2}\,c_d$
7	$\langle S\,u\cdot u\,u\cdot u\rangle$	$10/3\,N\,F^{-6}\,c_d^5+2/3\,N\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}-8/3\,N\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}+1/6\,N\,F^{-4}\,c_d^3-4/3\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2\,N\,F^{-2}\,c_d\,(\lambda_1^{\rm SS})^2+4/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}-1/24\,N\,F^{-2}\,c_d$
8	$\langle u_\nu S u^\nu\,u\cdot u\rangle$	$-2/3\,N\,F^{-6}\,c_d^5+2/3\,N\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+10/3\,N\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}-1/3\,N\,F^{-4}\,c_d^3-4\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}+2/3\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}+1/12\,N\,F^{-2}\,c_d$
9	$\langle S\rangle\,\langle u\cdot u\,u\cdot u\rangle$	$8/3\,F^{-6}\,c_d^5+4/3\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}-2\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}+1/2\,F^{-4}\,c_d^3+1/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2\,F^{-2}\,c_d\,(\lambda_1^{\rm SS})^2-F^{-2}\,c_d\,\lambda_2^{\rm SS}-1/24\,F^{-2}\,c_d$
10	$\langle u^\mu\rangle\,\langle u_\mu\,\{S,u\cdot u\}\rangle$	$4/3\,F^{-6}\,c_d^5-4/3\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+2\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}-3/2\,F^{-4}\,c_d^3-4F^{-2}\,c_d\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}+8/3F^{-2}\,c_d\,\lambda_1^{\rm SS}-2F^{-2}\,c_d\,(\lambda_1^{\rm SS})^2+1/24F^{-2}\,c_d$
11	$\langle u^\mu\,S\rangle\,\langle u_\mu\,u\cdot u\rangle$	$8\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}-2\,F^{-4}\,c_d^3-8\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}-4\,F^{-2}\,c_d\,(\lambda_1^{\rm SS})^2+2\,F^{-2}\,c_d\,\lambda_2^{\rm SS}+1/4\,F^{-2}\,c_d$
12	$\langle S\rangle\,\langle u_\mu\,u_\nu u^\mu u^\nu\rangle$	$-2/3\,F^{-6}\,c_d^5+2/3\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+1/2\,F^{-4}\,c_d^3-1/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-1/12\,F^{-2}\,c_d$
13	$\langle S\,u_\nu u^\mu u^\nu\rangle\,\langle u_\mu\rangle$	$-8/3\,F^{-6}\,c_d^5+8/3\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+4\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}+F^{-4}\,c_d^3-4/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}+2\,F^{-2}\,c_d\,\lambda_2^{\rm SS}-8\,F^{-2}\,c_d\,(\lambda_2^{\rm SS})^2-1/3\,F^{-2}\,c_d$
14	$\langle S\,u^\mu u^\nu\rangle\,\langle u_\mu\,u_\nu\rangle$	$-8\,F^{-6}\,c_d^5+8\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+16\,F^{-4}\,c_d^3\,\lambda_2^{\rm SS}+2\,F^{-4}\,c_d^3-8\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}-2\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2\,F^{-2}\,c_d\,\lambda_2^{\rm SS}-8\,F^{-2}\,c_d\,(\lambda_2^{\rm SS})^2$
15	$\langle S\,u\cdot u\rangle\,\langle u\cdot u\rangle$	$4\,F^{-6}\,c_d^5+4\,F^{-4}\,c_d^3\,\lambda_1^{\rm SS}+F^{-4}\,c_d^3-4\,F^{-2}\,c_d\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}-F^{-2}\,c_d\,\lambda_1^{\rm SS}-4\,F^{-2}\,c_d\,(\lambda_1^{\rm SS})^2-F^{-2}\,c_d\,\lambda_2^{\rm SS}$
16	$i\,\langle u_\mu f_+^{\mu\nu} u_\nu\,S\rangle$	$1/3\,N\,F^{-4}\,c_d^3-1/3\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}+1/12\,N\,F^{-2}\,c_d$
17	$i\,\left\langle\{S,u_\mu u_\nu\}\,f_+^{\mu\nu}\right\rangle$	$-1/6\,N\,F^{-4}\,c_d^3+1/6\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}+1/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}-1/24\,N\,F^{-2}\,c_d$
18	$i\,\langle u_\mu\rangle\,\left\langle f_+^{\mu\nu}\,[S,u_\nu]\right\rangle$	$-1/3\,F^{-4}\,c_d^3+1/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}+1/12\,F^{-2}\,c_d$
19	$i\,\langle S\rangle\,\left\langle f_+^{\mu\nu}\,u_\mu u_\nu\right\rangle$	$-2/3\,F^{-4}\,c_d^3+2/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}+1/6\,F^{-2}\,c_d$
20	$\left\langle f_-^{\mu\nu}\,\{u_\mu,\nabla_\nu S\}\right\rangle$	$1/3\,N\,F^{-4}\,c_d^3-1/3\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}+1/12\,N\,F^{-2}\,c_d$
21	$\langle u_\mu\rangle\,\left\langle f_-^{\mu\nu}\,\nabla_\nu S\right\rangle$	$2/3\,F^{-4}\,c_d^3-2/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-1/6\,F^{-2}\,c_d$
22	$\left\langle u_\mu\,f_-^{\mu\nu}\right\rangle\,\langle\nabla_\nu S\rangle$	$2/3\,F^{-4}\,c_d^3-2/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-1/6\,F^{-2}\,c_d$
23	$\left\langle S\,f_-^{\mu\nu}\,f_{-\,\mu\nu}\right\rangle$	$1/3\,N\,F^{-4}\,c_d^3-1/3\,N\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-2/3\,N\,F^{-2}\,c_d\,\lambda_2^{\rm SS}+1/12\,N\,F^{-2}\,c_d$
24	$\langle S\rangle\,\left\langle f_-^{\mu\nu}\,f_{-\,\mu\nu}\right\rangle$	$1/3\,F^{-4}\,c_d^3-1/3\,F^{-2}\,c_d\,\lambda_1^{\rm SS}-1/12\,F^{-2}\,c_d$

Table 2: (*continued*)

25	$\langle S\,\{\chi_+, u\cdot u\}\rangle$	$2NF^{-6}c_d^4c_m + NF^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 2NF^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} + 1/4\,NF^{-4}c_d^2c_m + NF^{-4}c_d^3\lambda_3^{\mathrm{SS}} + 1/2\,NF^{-4}c_d^3 - NF^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} - NF^{-2}c_d\lambda_1^{\mathrm{SS}} + 1/2\,NF^{-2}c_d\lambda_2^{\mathrm{SS}} - 1/4\,NF^{-2}c_d\lambda_3^{\mathrm{SS}} - 1/8\,NF^{-2}c_d - 1/2\,NF^{-2}c_m\lambda_1^{\mathrm{SS}} + 1/8\,NF^{-2}c_m$
26	$\langle u_\nu S u^\nu\,\chi_+\rangle$	$-2NF^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} + 4NF^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} - 1/2\,NF^{-4}c_d^2c_m + 2NF^{-4}c_d^3\lambda_3^{\mathrm{SS}} + 1/2\,NF^{-2}c_d\lambda_1^{\mathrm{SS}} - 4NF^{-2}c_d\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} - NF^{-2}c_d\lambda_2^{\mathrm{SS}} + 1/8\,NF^{-2}c_d$
27	$\langle S\,u\cdot u\rangle\langle\chi_+\rangle$	$4F^{-6}c_d^4c_m + 2F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 2F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} + 2F^{-4}c_d^3\lambda_3^{\mathrm{SS}} + F^{-4}c_d^3 - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} - 3/2\,F^{-2}c_d\lambda_1^{\mathrm{SS}} - 4F^{-2}c_d\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2\,F^{-2}c_d\lambda_3^{\mathrm{SS}} - 1/8\,F^{-2}c_d - F^{-2}c_m\lambda_2^{\mathrm{SS}} + 1/4\,F^{-2}c_m$
28	$\langle S\,\chi_+\rangle\langle u\cdot u\rangle$	$4F^{-6}c_d^4c_m - 4F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} + F^{-4}c_d^2c_m + 4F^{-4}c_d^3\lambda_3^{\mathrm{SS}} + F^{-4}c_d^3 - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2\,F^{-2}c_d\lambda_1^{\mathrm{SS}} - F^{-2}c_d\lambda_2^{\mathrm{SS}} - 3/2\,F^{-2}c_d\lambda_3^{\mathrm{SS}} - 3/8\,F^{-2}c_d - F^{-2}c_m\lambda_1^{\mathrm{SS}} + 1/4\,F^{-2}c_m$
29	$\langle u^\mu\rangle\langle u_\mu\,\{S,\chi_+\}\rangle$	$4F^{-6}c_d^4c_m - 4F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 2F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} - 1/2\,F^{-4}c_d^2c_m - F^{-4}c_d^3 - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2\,F^{-2}c_d\lambda_1^{\mathrm{SS}} - 2F^{-2}c_d\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2\,F^{-2}c_d\lambda_2^{\mathrm{SS}} + c_dF^{-2}\lambda_3^{\mathrm{SS}} + 1/4\,F^{-2}c_d + F^{-2}c_m\lambda_1^{\mathrm{SS}} - 1/4\,F^{-2}c_m$
30	$\langle u^\mu\,S\rangle\langle u_\mu\,\chi_+\rangle$	$8F^{-6}c_d^4c_m - 4F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 8F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} - F^{-4}c_d^2c_m + 4F^{-4}c_d^3\lambda_3^{\mathrm{SS}} - 2F^{-4}c_d^3 - 4F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + F^{-2}c_d\lambda_1^{\mathrm{SS}} - 4F^{-2}c_d\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + F^{-2}c_d\lambda_2^{\mathrm{SS}} + 1/2\,F^{-2}c_d + 2F^{-2}c_m\lambda_2^{\mathrm{SS}} - 1/2\,F^{-2}c_m$
31	$\langle S\rangle\langle\chi_+\,u\cdot u\rangle$	$4F^{-6}c_d^4c_m - 2F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 2F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} + F^{-4}c_d^2c_m + 2F^{-4}c_d^3\lambda_3^{\mathrm{SS}} + F^{-4}c_d^3 - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2\,F^{-2}c_d\lambda_1^{\mathrm{SS}} - F^{-2}c_d\lambda_2^{\mathrm{SS}} - 1/2\,F^{-2}c_d\lambda_3^{\mathrm{SS}} - 3/8\,F^{-2}c_d - F^{-2}c_m\lambda_2^{\mathrm{SS}} + 1/4\,F^{-2}c_m$
32	$i\,\langle\chi_-\,\{u^\mu,\nabla_\mu S\}\rangle$	$3NF^{-4}c_d^2c_m - 3/2\,NF^{-4}c_d^3 + 1/2\,NF^{-2}c_d\lambda_1^{\mathrm{SS}} + NF^{-2}c_d\lambda_2^{\mathrm{SS}} + 3/8\,NF^{-2}c_d - NF^{-2}c_m\lambda_1^{\mathrm{SS}} - 2NF^{-2}c_m\lambda_2^{\mathrm{SS}} - 3/4\,NF^{-2}c_m$
33	$i\,\langle u^\mu\,\chi_-\rangle\langle\nabla_\mu S\rangle$	$-2F^{-4}c_d^2c_m + F^{-4}c_d^3 + F^{-2}c_d\lambda_1^{\mathrm{SS}} - 3/4\,F^{-2}c_d - 2F^{-2}c_m\lambda_1^{\mathrm{SS}} + 3/2F^{-2}c_m$
34	$i\,\langle\chi_-\rangle\langle u^\mu\,\nabla_\mu S\rangle$	$6F^{-4}c_d^2c_m - 3F^{-4}c_d^3 + F^{-2}c_d\lambda_1^{\mathrm{SS}} + 2F^{-2}c_d\lambda_2^{\mathrm{SS}} + 3/4\,F^{-2}c_d - 2F^{-2}c_m\lambda_1^{\mathrm{SS}} - 4F^{-2}c_m\lambda_2^{\mathrm{SS}} - 3/2\,F^{-2}c_m$
35	$i\,\langle\chi_-\,\nabla_\mu S\rangle\langle u^\mu\rangle$	$-2F^{-4}c_d^2c_m + F^{-4}c_d^3 + F^{-2}c_d\lambda_1^{\mathrm{SS}} - 3/4\,F^{-2}c_d - 2F^{-2}c_m\lambda_1^{\mathrm{SS}} + 3/2\,F^{-2}c_m$
36	$\langle S\,\chi_+\,\chi_+\rangle$	$-NF^{-4}c_dc_m^2 + 4NF^{-4}c_d^2c_m\lambda_3^{\mathrm{SS}} + 2NF^{-4}c_d^2c_m - NF^{-2}c_d\lambda_3^{\mathrm{SS}} - 1/4\,NF^{-2}c_d - NF^{-2}c_m\lambda_1^{\mathrm{SS}} + 1/4\,NF^{-2}c_m$
37	$\langle S\rangle\langle\chi_+\,\chi_+\rangle$	$-F^{-4}c_dc_m^2 + 2F^{-4}c_d^2c_m - 1/4\,F^{-2}c_d - F^{-2}c_m\lambda_2^{\mathrm{SS}} + 1/4\,F^{-2}c_m$
38	$\langle S\,\chi_+\rangle\langle\chi_+\rangle$	$-2F^{-4}c_dc_m^2 + 4F^{-4}c_d^2c_m\lambda_3^{\mathrm{SS}} + 4F^{-4}c_d^2c_m - F^{-2}c_d\lambda_3^{\mathrm{SS}} - 1/2\,F^{-2}c_d - F^{-2}c_m\lambda_1^{\mathrm{SS}} - F^{-2}c_m\lambda_2^{\mathrm{SS}} + 1/2\,F^{-2}c_m$
39	$\langle S\,\chi_-\,\chi_-\rangle$	$6NF^{-4}c_dc_m^2 - 8NF^{-4}c_d^2c_m + 5/2\,NF^{-4}c_d^3 - 1/2\,NF^{-2}c_d\lambda_1^{\mathrm{SS}} - NF^{-2}c_d\lambda_2^{\mathrm{SS}} + NF^{-2}c_d\lambda_3^{\mathrm{SS}} - 1/8\,NF^{-2}c_d + NF^{-2}c_m\lambda_1^{\mathrm{SS}} + 2F^{-2}Nc_m\lambda_2^{\mathrm{SS}} - 2NF^{-2}c_m\lambda_3^{\mathrm{SS}} + 1/4\,NF^{-2}c_m$
40	$\langle S\rangle\langle\chi_-\,\chi_-\rangle$	$2F^{-4}c_dc_m^2 - 2F^{-4}c_d^2c_m + 1/2\,F^{-4}c_d^3 - 1/2\,F^{-2}c_d\lambda_1^{\mathrm{SS}} + F^{-2}c_d\lambda_3^{\mathrm{SS}} + 1/8\,F^{-2}c_d + F^{-2}c_m\lambda_1^{\mathrm{SS}} - 2F^{-2}c_m\lambda_3^{\mathrm{SS}} - 1/4\,F^{-2}c_m$
41	$\langle S\,\chi_-\rangle\langle\chi_-\rangle$	$8F^{-4}c_dc_m^2 - 10F^{-4}c_d^2c_m + 3F^{-4}c_d^3 - F^{-2}c_d\lambda_1^{\mathrm{SS}} - F^{-2}c_d\lambda_2^{\mathrm{SS}} + 2F^{-2}c_d\lambda_3^{\mathrm{SS}} + 2F^{-2}c_m\lambda_1^{\mathrm{SS}} + 2F^{-2}c_m\lambda_2^{\mathrm{SS}} - 4F^{-2}c_m\lambda_3^{\mathrm{SS}}$

Table 2: Operators with one scalar resonance and their β-function coefficients when only scalar resonances are included.

i	$\mathcal{O}_i^{SS}$	γ_i^{SS}
1	$\langle SS\rangle$	$N\,F^{-4}\,c_d^2\,M_S^4$
2	$\langle S\rangle^2$	$F^{-4}\,c_d^2\,M_S^4$
3	$\langle SS\,u\cdot u\rangle$	$-4\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,M_S^2 - 4\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,M_S^2 - N\,F^{-4}\,c_d^2\,M_S^2 + 1/2\,N\,F^{-2}\,\lambda_1^{\rm SS}\,M_S^2 + N\,F^{-2}\,(\lambda_1^{\rm SS})^2\,M_S^2 + 4\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2\,M_S^2 + 1/16\,N\,F^{-2}\,M_S^2$
4	$\langle u_\nu S u^\nu\,S\rangle$	$-2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,M_S^2 - 4\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,M_S^2 + 3/2\,N\,F^{-4}\,c_d^2\,M_S^2 + 4\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,M_S^2 - N\,F^{-2}\,\lambda_2^{\rm SS}\,M_S^2$
5	$\langle u^\mu\rangle\,\langle u_\mu\,SS\rangle$	$-4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,M_S^2 - 4\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,M_S^2 + 2\,F^{-4}\,c_d^2\,M_S^2 + 4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,M_S^2 - 2\,F^{-2}\,\lambda_1^{\rm SS}\,M_S^2 + 2\,F^{-2}\,(\lambda_1^{\rm SS})^2\,M_S^2 + F^{-2}\,\lambda_2^{\rm SS}\,M_S^2 - 1/8\,F^{-2}\,M_S^2$
6	$\langle u^\mu\,S\rangle^2$	$-4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,M_S^2 - 4\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,M_S^2 + 2\,F^{-4}\,c_d^2\,M_S^2 + 4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,M_S^2 + 2\,F^{-2}\,(\lambda_1^{\rm SS})^2\,M_S^2 - 3\,F^{-2}\,\lambda_2^{\rm SS}\,M_S^2 + 4\,F^{-2}\,(\lambda_2^{\rm SS})^2\,M_S^2 + 1/8\,F^{-2}\,M_S^2$
7	$\langle S\rangle\,\langle S\,u\cdot u\rangle$	$-8\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,M_S^2 - 8\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,M_S^2 - 2\,F^{-4}\,c_d^2\,M_S^2 + 4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,M_S^2 + 2\,F^{-2}\,(\lambda_1^{\rm SS})^2\,M_S^2 + 3\,F^{-2}\,\lambda_2^{\rm SS}\,M_S^2 - 1/8\,F^{-2}\,M_S^2$
8	$\langle SS\rangle\,\langle u\cdot u\rangle$	$-2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,M_S^2 - 5/2\,F^{-4}\,c_d^2\,M_S^2 + 3/2\,F^{-2}\,\lambda_1^{\rm SS}\,M_S^2 + F^{-2}\,(\lambda_1^{\rm SS})^2\,M_S^2 + 1/16\,F^{-2}\,M_S^2$
9	$\langle SS\,\chi_+\rangle$	$N\,F^{-4}\,c_d\,c_m\,M_S^2 - 4\,N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,M_S^2 - 2\,N\,F^{-4}\,c_d^2\,M_S^2 + N\,F^{-2}\,\lambda_1^{\rm SS}\,M_S^2$
10	$\langle S\rangle\,\langle S\,\chi_+\rangle$	$2\,F^{-4}\,c_d\,c_m\,M_S^2 - 4\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,M_S^2 - 4\,F^{-4}\,c_d^2\,M_S^2 + 2\,F^{-2}\,\lambda_2^{\rm SS}\,M_S^2$
11	$\langle SS\rangle\,\langle\chi_+\rangle$	$F^{-4}\,c_d\,c_m\,M_S^2 - 2\,F^{-4}\,c_d^2\,M_S^2 + F^{-2}\,\lambda_1^{\rm SS}\,M_S^2$
12	$\langle u^\mu\,u^\nu\,S\,u_\mu\,u_\nu\,S\rangle$	$2/3\,N\,F^{-8}\,c_d^6 - 8/3\,N\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} - 1/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 2/3\,N\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 + 1/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 8/3\,N\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 - 1/24\,N\,F^{-4}\,c_d^2 + 1/24\,N\,F^{-2}\,\lambda_1^{\rm SS} - 1/12\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 1/3\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 + 1/64\,NF^{-2}$
13	$\langle S\,u_\mu\,S\,u_\nu u^\mu u^\nu\rangle$	$-8/3\,N\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} + 2/3\,N\,F^{-6}\,c_d^4 + 4\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 2/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 1/6\,N\,F^{-4}\,c_d^2 - 2/3\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 1/6\,F^{-2}\,N\,\lambda_2^{\rm SS}$
14	$\langle S\,u\cdot u\,S\,u\cdot u\rangle$	$2/3\,N\,F^{-8}\,c_d^6 + 4\,N\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} - 8/3\,N\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} - 4\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 2/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 2/3\,N\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 - N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 8/3\,N\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 - 1/12\,N\,F^{-4}\,c_d^2 - 1/16\,N\,F^{-2}\,\lambda_1^{\rm SS} - 1/2\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 - N\,F^{-2}\,(\lambda_1^{\rm SS})^3 + N\,F^{-2}\,(\lambda_2^{\rm SS})^2$
15	$\langle SS\,u_\mu u_\nu u^\mu u^\nu\rangle$	$4/3\,N\,F^{-8}\,c_d^6 - 2/3\,N\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} - 8/3\,N\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} - 2/3\,N\,F^{-6}\,c_d^4 + 1/6\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 1/3\,N\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 + N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 4/3\,N\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 + 5/48\,N\,F^{-4}\,c_d^2 + 1/24\,N\,F^{-2}\,\lambda_1^{\rm SS} - 1/12\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 1/3\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 - 5/192\,NF^{-2}$
16	$\langle SS\,u\cdot u\,u\cdot u\rangle$	$4/3\,N\,F^{-8}\,c_d^6 + 10/3\,N\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} - 8/3\,N\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} + 4/3\,N\,F^{-6}\,c_d^4 - 4\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 1/6\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 1/3\,N\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 - 5/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 4/3\,N\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 + 7/48\,N\,F^{-4}\,c_d^2 - 1/24\,N\,F^{-2}\,\lambda_1^{\rm SS} - 11/12\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 1/3\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 + 1/192\,N\,F^{-2}$

Table 3: (*continued*)

17	$\langle u_\nu S u^\nu \ \{S, u\cdot u\}\rangle$	$-8/3\,N F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} + 4\,N F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} - 1/3\,N F^{-6} c_d^4 + 6\,N F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 11/6\,N F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 3/2\,N F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} - 4\,N F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 - 1/24\,N F^{-4} c_d^2 + 1/3\,N F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 2\,N F^{-2} (\lambda_1^{\mathrm{SS}})^2 \lambda_2^{\mathrm{SS}} + 1/24\,N F^{-2} \lambda_2^{\mathrm{SS}}$
18	$\langle u_\mu S u^\mu \, u_\nu S u^\nu\rangle$	$2/3\,N F^{-8} c_d^6 - 8/3\,N F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} - 4\,N F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 2/3\,N F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 2/3\,N F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 - 4/3\,N F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} + 20/3\,N F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 + 1/8\,N F^{-4} c_d^2 - 1/24\,N F^{-2} \lambda_1^{\mathrm{SS}} + 1/12\,N F^{-2} (\lambda_1^{\mathrm{SS}})^2 + 1/3\,N F^{-2} (\lambda_2^{\mathrm{SS}})^2 - 1/64\,N F^{-2}$
19	$\langle u_\nu S S u^\nu \, u\cdot u\rangle$	$4/3\,N F^{-8} c_d^6 - 14/3\,N F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 8/3\,N F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} + 1/3\,N F^{-6} c_d^4 + 8\,N F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 1/6\,N F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 7/3\,N F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 - 1/3\,N F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} + 4/3\,N F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 - 3/16\,N F^{-4} c_d^2 - 4\,N F^{-2} \lambda_1^{\mathrm{SS}} (\lambda_2^{\mathrm{SS}})^2 + 1/8\,N F^{-2} \lambda_1^{\mathrm{SS}} + 1/4\,N F^{-2} (\lambda_1^{\mathrm{SS}})^2 - 5/192\,N F^{-2}$
20	$\langle u^\mu\rangle \langle u_\mu \ \{SS, u\cdot u\}\rangle$	$1/24 F^{-2} + 16/3\,F^{-8} c_d^6 - 16/3\,F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 8\,F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} + 1/3\,F^{-6} c_d^4 + 14/3\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 11/6\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 4/3\,F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 + 1/3\,F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} + 8/3\,F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 - 1/6\,F^{-4} c_d^2 - 1/3\,F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 2\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 \lambda_2^{\mathrm{SS}} + 4/3\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 + 1/24\,F^{-2} \lambda_2^{\mathrm{SS}}$
21	$\langle u^\mu\rangle \langle u_\mu \ \{u_\nu S u^\nu, S\}\rangle$	$8/3\,F^{-8} c_d^6 - 8\,F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 2/3\,F^{-6} c_d^4 + 16/3\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 4/3\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 8/3\,F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 - 2\,F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} + 4/3\,F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 + 1/4\,F^{-4} c_d^2 + F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 4\,F^{-2} \lambda_1^{\mathrm{SS}} (\lambda_2^{\mathrm{SS}})^2 - 1/4\,F^{-2} \lambda_2^{\mathrm{SS}} + F^{-2} (\lambda_2^{\mathrm{SS}})^2$
22	$\langle u^\mu \, S\rangle \langle u_\mu \ \{S, u\cdot u\}\rangle$	$-1/12 F^{-2} + 16\,F^{-8} c_d^6 - 64/3\,F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 80/3\,F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} + 8/3\,F^{-6} c_d^4 + 24\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 14/3\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 12\,F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 - 8/3\,F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} + 8\,F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 - 7/12\,F^{-4} c_d^2 + 3\,F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 4\,F^{-2} \lambda_1^{\mathrm{SS}} (\lambda_2^{\mathrm{SS}})^2 - 1/24\,F^{-2} \lambda_1^{\mathrm{SS}} - 6\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 \lambda_2^{\mathrm{SS}} + 2/3\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 - 2\,F^{-2} (\lambda_1^{\mathrm{SS}})^3 + 5/8\,F^{-2} \lambda_2^{\mathrm{SS}} + F^{-2} (\lambda_2^{\mathrm{SS}})^2$
23	$\langle u^\mu \, S \, u^\nu \, S\rangle \langle u_\mu \, u_\nu\rangle$	$16/3\,F^{-8} c_d^6 - 16\,F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 32/3\,F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} + 4/3\,F^{-6} c_d^4 + 64/3\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 2\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 16/3\,F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 - 4/3\,F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} + 8/3\,F^{-4} c_d^2 (\lambda_2^{\mathrm{SS}})^2 - 2/3\,F^{-4} c_d^2 - 2\,F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 4\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 \lambda_2^{\mathrm{SS}} + 3/4\,F^{-2} \lambda_2^{\mathrm{SS}}$
24	$\langle u^\mu \, SS\rangle \langle u_\mu \, u\cdot u\rangle$	$3/16 F^{-2} + 24\,F^{-8} c_d^6 - 48\,F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 24\,F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} + 32\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 24\,F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 - 1/2\,F^{-4} c_d^2 - F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 3/8\,F^{-2} \lambda_1^{\mathrm{SS}} - 6\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 \lambda_2^{\mathrm{SS}} - 2\,F^{-2} (\lambda_1^{\mathrm{SS}})^3 - 3/8\,F^{-2} \lambda_2^{\mathrm{SS}}$
25	$\langle S\rangle \langle S \, u_\mu u_\nu u^\mu u^\nu\rangle$	$1/48 F^{-2} + 4\,F^{-8} c_d^6 - 16/3\,F^{-6} c_d^4 \lambda_1^{\mathrm{SS}} - 20/3\,F^{-6} c_d^4 \lambda_2^{\mathrm{SS}} - 4/3\,F^{-6} c_d^4 + 4\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} + 4/3\,F^{-4} c_d^2 \lambda_1^{\mathrm{SS}} + 2\,F^{-4} c_d^2 (\lambda_1^{\mathrm{SS}})^2 + 3\,F^{-4} c_d^2 \lambda_2^{\mathrm{SS}} - 1/8\,F^{-4} c_d^2 - 2/3\,F^{-2} \lambda_1^{\mathrm{SS}} \lambda_2^{\mathrm{SS}} - 1/3\,F^{-2} (\lambda_1^{\mathrm{SS}})^2 - 1/6\,F^{-2} \lambda_2^{\mathrm{SS}}$

Table 3: (*continued*)

26	$\langle S\rangle\,\langle S\,u\cdot u\,u\cdot u\rangle$	$-1/96F^{-2} + 4\,F^{-8}\,c_d^6 + 8/3\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} - 20/3\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} + 14/3\,F^{-6}\,c_d^4 + 1/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 2\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 - 6\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 1/8\,F^{-4}\,c_d^2 - 2/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 1/8\,F^{-2}\,\lambda_1^{\mathrm{SS}} + 1/6\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^3 - 1/6\,F^{-2}\,\lambda_2^{\mathrm{SS}}$
27	$\langle S\,u_\mu\,S\,u\cdot u\rangle\,\langle u^\mu\rangle$	$8/3\,F^{-8}\,c_d^6 - 8\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} + 4/3\,F^{-6}\,c_d^4 + 16/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 14/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 4/3\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 - F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 16/3\,F^{-4}\,c_d^2\,(\lambda_2^{\mathrm{SS}})^2 + 1/2\,F^{-4}\,c_d^2 + 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 1/8\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2\,\lambda_2^{\mathrm{SS}} + 2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^3 - 3/8\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 2\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2$
28	$\langle S\,u_\nu u^\mu u^\nu\rangle\,\langle u_\mu\,S\rangle$	$-5/24F^{-2} + 16\,F^{-8}\,c_d^6 - 64/3\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} - 80/3\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} - 16/3\,F^{-6}\,c_d^4 + 16\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 16/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 8\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 + 4/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 16\,F^{-4}\,c_d^2\,(\lambda_2^{\mathrm{SS}})^2 + 7/6\,F^{-4}\,c_d^2 - 8\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,(\lambda_2^{\mathrm{SS}})^2 - 1/6\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 4/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 2\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2$
29	$\langle S\;[u^\mu,u^\nu]\rangle^2$	$3/64F^{-2} - 4\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} + F^{-6}\,c_d^4 + 6\,F^{-4}\,c_d^2\,(\lambda_2^{\mathrm{SS}})^2 - 3/8\,F^{-4}\,c_d^2 + 1/2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,(\lambda_2^{\mathrm{SS}})^2 - F^{-2}\,(\lambda_1^{\mathrm{SS}})^2\,\lambda_2^{\mathrm{SS}} - 1/4\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 1/16\,F^{-2}\,\lambda_2^{\mathrm{SS}} - F^{-2}\,(\lambda_2^{\mathrm{SS}})^2 - 2\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^3$
30	$\langle S\;\{u^\mu,u^\nu\}\rangle^2$	$3/64F^{-2} + 6\,F^{-8}\,c_d^6 - 8\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} - 14\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} - F^{-6}\,c_d^4 + 10\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 3\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 + 3/2\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 10\,F^{-4}\,c_d^2\,(\lambda_2^{\mathrm{SS}})^2 - 1/16\,F^{-4}\,c_d^2 - 1/2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,(\lambda_2^{\mathrm{SS}})^2 - F^{-2}\,(\lambda_1^{\mathrm{SS}})^2\,\lambda_2^{\mathrm{SS}} - 1/4\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 3/16\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 2\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^3$
31	$\langle S\,u\cdot u\rangle^2$	$3/32F^{-2} + 12\,F^{-8}\,c_d^6 - 16\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} - 12\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} + 2\,F^{-6}\,c_d^4 + 16\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 14\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 - 2\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 3/8\,F^{-4}\,c_d^2 - 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 1/8\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 4\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2\,\lambda_2^{\mathrm{SS}} + 1/2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^3 - 1/4\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 2\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2$
32	$\langle SS\rangle\,\langle u_\mu\,u_\nu u^\mu u^\nu\rangle$	$-1/96F^{-2} + 2\,F^{-8}\,c_d^6 - 10/3\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} - 4/3\,F^{-6}\,c_d^4 + 3/2\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 + 11/48\,F^{-4}\,c_d^2 - 1/12\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/6\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2$
33	$\langle SS\rangle\,\langle u\cdot u\,u\cdot u\rangle$	$-7/192F^{-2} + 4\,F^{-8}\,c_d^6 - 32/3\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} + 7/3\,F^{-6}\,c_d^4 - 2\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 8\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 + 1/12\,F^{-4}\,c_d^2 - 5/48\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/12\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - F^{-2}\,(\lambda_1^{\mathrm{SS}})^3$
34	$\langle SS\,u_\nu u^\mu u^\nu\rangle\,\langle u_\mu\rangle$	$5/48F^{-2} + 16/3\,F^{-8}\,c_d^6 - 16/3\,F^{-6}\,c_d^4\,\lambda_1^{\mathrm{SS}} - 16\,F^{-6}\,c_d^4\,\lambda_2^{\mathrm{SS}} - 2/3\,F^{-6}\,c_d^4 + 20/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 5/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 4/3\,F^{-4}\,c_d^2\,(\lambda_1^{\mathrm{SS}})^2 + 1/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 56/3\,F^{-4}\,c_d^2\,(\lambda_2^{\mathrm{SS}})^2 - 1/6\,F^{-4}\,c_d^2 - 1/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 1/4\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 2/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 1/12\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 8\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^3$

Table 3: (*continued*)

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35	$\langle SS\,u^\mu u_\nu\rangle\,\langle u_\mu\,u^\nu\rangle$	$-3/16F^{-2} + 56/3\,F^{-8}\,c_d^6 - 24\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} - 112/3\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} - 16/3\,F^{-6}\,c_d^4 + 80/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 20/3\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 + 16/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 64/3\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 + 11/12\,F^{-4}\,c_d^2 - 8\,F^{-2}\,\lambda_1^{\rm SS}\,(\lambda_2^{\rm SS})^2 - F^{-2}\,(\lambda_1^{\rm SS})^2 - 2\,F^{-2}\,(\lambda_2^{\rm SS})^2$
36	$\langle SS\,u\cdot u\rangle\,\langle u\cdot u\rangle$	$-3/32F^{-2} + 28/3\,F^{-8}\,c_d^6 - 12\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} - 32/3\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} + 10/3\,F^{-6}\,c_d^4 + 40/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 10/3\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 - 10/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 8/3\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 - 1/24\,F^{-4}\,c_d^2 - 4\,F^{-2}\,\lambda_1^{\rm SS}\,(\lambda_2^{\rm SS})^2 - 1/8\,F^{-2}\,\lambda_1^{\rm SS} - 3/2\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 2\,F^{-2}\,(\lambda_1^{\rm SS})^3 + F^{-2}\,(\lambda_2^{\rm SS})^2$
37	$\langle u_\nu S u^\nu\,u\cdot u\rangle\,\langle S\rangle$	$1/12F^{-2} + 4\,F^{-8}\,c_d^6 - 40/3\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} + 4/3\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} - 4/3\,F^{-6}\,c_d^4 + 12\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 2/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 6\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 + 3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 4\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 - 1/8\,F^{-4}\,c_d^2 - 2/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 4\,F^{-2}\,(\lambda_1^{\rm SS})^2\,\lambda_2^{\rm SS} + 2/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 1/12\,F^{-2}\,\lambda_2^{\rm SS} - 2\,F^{-2}\,(\lambda_2^{\rm SS})^2$
38	$\langle u_\nu S u^\nu\,S\rangle\,\langle u\cdot u\rangle$	$8/3\,F^{-8}\,c_d^6 - 8\,F^{-6}\,c_d^4\,\lambda_1^{\rm SS} + 8/3\,F^{-6}\,c_d^4\,\lambda_2^{\rm SS} - 4/3\,F^{-6}\,c_d^4 - 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 20/3\,F^{-4}\,c_d^2\,(\lambda_1^{\rm SS})^2 + 13/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 4/3\,F^{-4}\,c_d^2\,(\lambda_2^{\rm SS})^2 - 1/3\,F^{-4}\,c_d^2 - 4\,F^{-2}\,(\lambda_1^{\rm SS})^2\,\lambda_2^{\rm SS} + 1/4\,F^{-2}\,\lambda_2^{\rm SS}$
39	$\langle u^\mu\,u^\nu\,\nabla_\mu S\,\nabla_\nu S\rangle$	$2/3\,N\,F^{-6}\,c_d^4 - 4/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 1/3\,N\,F^{-4}\,c_d^2 - 1/6\,N\,F^{-2}\,\lambda_1^{\rm SS} + 1/3\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 4/3\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 + 5/48\,F^{-2}\,N$
40	$\langle u^\mu\,u^\nu\,\nabla_\nu S\,\nabla_\mu S\rangle$	$14/3\,N\,F^{-6}\,c_d^4 - 4\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 4\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 8/3\,N\,F^{-4}\,c_d^2 + 3/2\,N\,F^{-2}\,\lambda_1^{\rm SS} + N\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 4\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 + 23/48\,N\,F^{-2}$
41	$\langle u^\mu\,\nabla_\mu S\,u^\nu\,\nabla_\nu S\rangle + \text{h.c.}$	$2\,N\,F^{-6}\,c_d^4 - 4/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 4\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 2/3\,N\,F^{-4}\,c_d^2 + 8/3\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 4/3\,N\,F^{-2}\,\lambda_2^{\rm SS}$
42	$\langle\nabla_\mu S\,u_\nu\nabla^\mu S u^\nu\rangle$	$2/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 1/6\,N\,F^{-4}\,c_d^2 - 4/3\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 1/3\,N\,F^{-2}\,\lambda_2^{\rm SS}$
43	$\langle u\cdot u\,\nabla^\mu S\,\nabla_\mu S\rangle$	$2/3\,N\,F^{-6}\,c_d^4 - 2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 4/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 1/2\,N\,F^{-4}\,c_d^2 - 5/6\,N\,F^{-2}\,\lambda_1^{\rm SS} - 1/3\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 4/3\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 - 1/48\,N\,F^{-2}$
44	$\langle u_\mu\rangle\,\langle u^\mu\,\nabla^\nu S\,\nabla_\nu S\rangle$	$1/24F^{-2} + 4/3\,F^{-6}\,c_d^4 - 8/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 4/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - F^{-4}\,c_d^2 - 4/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 2\,F^{-2}\,\lambda_1^{\rm SS} - 2/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 1/3\,F^{-2}\,\lambda_2^{\rm SS}$
45	$\langle u^\mu\,u^\nu\rangle\,\langle\nabla_\mu S\,\nabla_\nu S\rangle$	$7/12F^{-2} + 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 5/3\,F^{-4}\,c_d^2 - 4/3\,F^{-2}\,\lambda_1^{\rm SS} + 4/3\,F^{-2}\,(\lambda_1^{\rm SS})^2$
46	$\langle u^\mu\,\nabla_\mu S\rangle\,\langle u^\nu\,\nabla_\nu S\rangle$	$7/12F^{-2} + 28/3\,F^{-6}\,c_d^4 - 20/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 12\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 14/3\,F^{-4}\,c_d^2 + 4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 5/3\,F^{-2}\,\lambda_1^{\rm SS} + 4/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 3\,F^{-2}\,\lambda_2^{\rm SS} + 4\,F^{-2}\,(\lambda_2^{\rm SS})^2$
47	$\langle u^\mu\,\nabla_\nu S\rangle^2$	$-1/24F^{-2} + 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 8/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - F^{-4}\,c_d^2 - 4/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 2/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 7/3\,F^{-2}\,\lambda_2^{\rm SS} - 4/3\,F^{-2}\,(\lambda_2^{\rm SS})^2$

Table 3: (*continued*)

48	$\langle u^\mu \nabla_\nu S\rangle \langle u^\nu \nabla_\mu S\rangle$	$7/12F^{-2} + 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 4/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 4/3\,F^{-4}\,c_d^2 + 4/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 5/3\,F^{-2}\,\lambda_1^{\mathrm{SS}} + 4/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 1/3\,F^{-2}\,\lambda_2^{\mathrm{SS}} + 4/3\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2$
49	$\langle u^\mu \,\{\nabla_\mu S, \nabla_\nu S\}\rangle \langle u^\nu\rangle$	$-7/12F^{-2} - 8/3\,F^{-6}\,c_d^4 - 8/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 4/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 3\,F^{-4}\,c_d^2 + 8/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 4/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 4/3\,F^{-2}\,\lambda_2^{\mathrm{SS}}$
50	$\langle \nabla^\mu S\rangle \langle \nabla_\mu S\, u\cdot u\rangle$	$1/24F^{-2} + 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 8/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + F^{-4}\,c_d^2 - 4/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 2/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 7/3\,F^{-2}\,\lambda_2^{\mathrm{SS}}$
51	$\langle \nabla_\mu S\,\{u^\mu, u^\nu\}\rangle \langle \nabla_\nu S\rangle$	$-7/12F^{-2} - 8/3\,F^{-6}\,c_d^4 - 8/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 4/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 3\,F^{-4}\,c_d^2 + 8/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 4/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 4/3\,F^{-2}\,\lambda_2^{\mathrm{SS}}$
52	$\langle u\cdot u\rangle \langle \nabla^\mu S\,\nabla_\mu S\rangle$	$-1/48F^{-2} + 2/3\,F^{-6}\,c_d^4 - 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 2/3\,F^{-4}\,c_d^2 - 7/6\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2$
53	$i\,\langle u_\mu\, u_\nu\, SS\, f_+^{\mu\nu}\rangle$ + h.c.	$1/3\,N\,F^{-6}\,c_d^4 - 1/6\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 1/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 1/8\,N\,F^{-4}\,c_d^2 + 1/24\,N\,F^{-2}$
54	$i\,\langle u_\mu\, S\, u_\nu\, S\, f_+^{\mu\nu}\rangle$ + h.c.	$-2/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 1/6\,N\,F^{-4}\,c_d^2 + 2/3\,N\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 1/6\,N\,F^{-2}\,\lambda_2^{\mathrm{SS}}$
55	$i\,\langle S\, u_\mu\, u_\nu\, S\, f_+^{\mu\nu}\rangle$	$-2/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 1/4\,N\,F^{-4}\,c_d^2 - 1/12\,N\,F^{-2}\,\lambda_1^{\mathrm{SS}} + 1/6\,N\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 2/3\,N\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2 - 7/96\,N\,F^{-2}$
56	$i\,\langle u_\mu f_+^{\mu\nu} u_\nu\, SS\rangle$	$-2/3\,N\,F^{-6}\,c_d^4 + 1/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 4/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 1/12\,N\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/6\,N\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 2/3\,N\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2 - 1/96\,N\,F^{-2}$
57	$i\,\langle u_\mu\rangle \langle f_+^{\mu\nu}\,[SS, u_\nu]\rangle$	$2/3\,F^{-6}\,c_d^4 - F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 1/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 1/12\,F^{-4}\,c_d^2 + 1/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 1/6\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 1/12\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 1/96F^{-2}$
58	$i\,\langle u_\mu\, S\rangle \langle f_+^{\mu\nu}\,[S, u_\nu]\rangle$	$4/3\,F^{-6}\,c_d^4 - 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 4/3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 1/6\,F^{-4}\,c_d^2 + 1/6\,F^{-2}\,\lambda_1^{\mathrm{SS}} + 1/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 1/48F^{-2}$
59	$i\,\langle S\rangle \langle u_\mu\, S\, f_+^{\mu\nu}\, u_\nu\rangle$ + h.c.	$-2/3\,F^{-6}\,c_d^4 + 2/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 1/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 1/6\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 1/12\,F^{-2}\,\lambda_2^{\mathrm{SS}} + 1/96F^{-2}$
60	$i\,\langle SS\rangle \langle f_+^{\mu\nu}\, u_\mu u_\nu\rangle$	$4/3\,F^{-6}\,c_d^4 - 2\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 1/3\,F^{-4}\,c_d^2 + 1/6\,F^{-2}\,\lambda_1^{\mathrm{SS}} + 1/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 1/48F^{-2}$
61	$i\,\langle u_\mu f_+^{\mu\nu} u_\nu\, S\rangle \langle S\rangle$	$-4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 2\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 2/3\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} - 1/3\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 1/6\,F^{-2}\,\lambda_2^{\mathrm{SS}} + 1/48F^{-2}$
62	$i\,\langle f_+^{\mu\nu}\,\nabla_\mu S\,\nabla_\nu S\rangle$	$1/3\,N\,F^{-4}\,c_d^2 - 1/6\,N\,F^{-2}$
63	$\langle u_\mu\, f_-^{\mu\nu}\,\nabla_\nu S\, S\rangle$ + h.c.	$-1/2\,N\,F^{-6}\,c_d^4 + 1/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 2/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}}$
64	$\langle u_\mu\,\nabla_\nu S\, S\, f_-^{\mu\nu}\rangle$ + h.c.	$-5/6\,N\,F^{-6}\,c_d^4 + 1/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 2\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} + 1/6\,N\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/3\,N\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 4/3\,N\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2 - 1/48\,N\,F^{-2}$
65	$\langle u_\mu\, S\, f_-^{\mu\nu}\,\nabla_\nu S\rangle$ + h.c.	$4/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 1/3\,N\,F^{-4}\,c_d^2 - 4/3\,N\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 1/3\,N\,F^{-2}\,\lambda_2^{\mathrm{SS}}$

Table 3: *(continued)*

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66	$\langle u_\mu\rangle \left\langle f_-^{\mu\nu}\ \{S,\nabla_\nu S\}\right\rangle$	$1/48F^{-2} - 4/3\,F^{-6}\,c_d^4 + 2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 2/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 1/6\,F^{-4}\,c_d^2 - 2/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 1/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 1/6\,F^{-2}\,\lambda_2^{\rm SS}$
67	$\langle u_\mu\, S\rangle \left\langle f_-^{\mu\nu}\,\nabla_\nu S\right\rangle$	$-1/24F^{-2} - 8/3\,F^{-6}\,c_d^4 + 8/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 8/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 1/3\,F^{-4}\,c_d^2 - 1/3\,F^{-2}\,\lambda_1^{\rm SS} - 2/3\,F^{-2}\,(\lambda_1^{\rm SS})^2$
68	$\langle \nabla_\mu S\rangle \left\langle f_-^{\mu\nu}\ \{S,u_\nu\}\right\rangle$	$-1/48F^{-2} + 4/3\,F^{-6}\,c_d^4 - 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 2\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 2/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 1/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 1/6\,F^{-2}\,\lambda_2^{\rm SS}$
69	$\langle S\rangle \left\langle f_-^{\mu\nu}\ \{u_\mu,\nabla_\nu S\}\right\rangle$	$1/48F^{-2} - 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 2\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 2/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 1/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 1/6\,F^{-2}\,\lambda_2^{\rm SS}$
70	$\left\langle S\, f_-^{\mu\nu}\right\rangle \langle u_\mu\,\nabla_\nu S\rangle$	$-1/24F^{-2} - 8/3\,F^{-6}\,c_d^4 + 8/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 16/3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 1/3\,F^{-4}\,c_d^2 - 8/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 1/3\,F^{-2}\,\lambda_1^{\rm SS} - 2/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 2/3\,F^{-2}\,\lambda_2^{\rm SS} - 8/3\,F^{-2}\,(\lambda_2^{\rm SS})^2$
71	$\langle S\,\nabla_\mu S\rangle \left\langle u_\nu\, f_-^{\mu\nu}\right\rangle$	$1/24F^{-2} + 8/3\,F^{-6}\,c_d^4 - 4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 2/3\,F^{-4}\,c_d^2 + 1/3\,F^{-2}\,\lambda_1^{\rm SS} + 2/3\,F^{-2}\,(\lambda_1^{\rm SS})^2$
72	$\left\langle S\, f_+^{\mu\nu}\, S\, f_{+\,\mu\nu}\right\rangle$	$1/12\,N\,F^{-4}\,c_d^2 - 1/24\,N\,F^{-2}$
73	$\left\langle SS\, f_+^{\mu\nu}\, f_{+\,\mu\nu}\right\rangle$	$-1/12\,N\,F^{-4}\,c_d^2 + 1/24\,N\,F^{-2}$
74	$\left\langle S\, f_-^{\mu\nu}\, S\, f_{-\,\mu\nu}\right\rangle$	$2/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 1/6\,N\,F^{-4}\,c_d^2 - 2/3\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} + 1/6\,N\,F^{-2}\,\lambda_2^{\rm SS}$
75	$\left\langle SS\, f_-^{\mu\nu}\, f_{-\,\mu\nu}\right\rangle$	$-2/3\,N\,F^{-6}\,c_d^4 + 1/3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 4/3\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 1/12\,N\,F^{-2}\,\lambda_1^{\rm SS} - 1/6\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 2/3\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2 - 1/96\,N\,F^{-2}$
76	$\langle S\rangle \left\langle S\, f_-^{\mu\nu}\, f_{-\,\mu\nu}\right\rangle$	$1/48F^{-2} - 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 2\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 2/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 1/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 1/6\,F^{-2}\,\lambda_2^{\rm SS}$
77	$\left\langle S\, f_-^{\mu\nu}\right\rangle^2$	$-1/48F^{-2} - 4/3\,F^{-6}\,c_d^4 + 4/3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 2\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 2/3\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 1/3\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 1/6\,F^{-2}\,\lambda_2^{\rm SS} - 2/3\,F^{-2}\,(\lambda_2^{\rm SS})^2$
78	$\langle SS\rangle \left\langle f_-^{\mu\nu}\, f_{-\,\mu\nu}\right\rangle$	$-1/96F^{-2} - 2/3\,F^{-6}\,c_d^4 + F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 1/6\ F^{-4}\,c_d^2 - 1/12\,F^{-2}\,\lambda_1^{\rm SS} - 1/6\,F^{-2}\,(\lambda_1^{\rm SS})^2$
79	$\langle S\,\chi_+\, S\, u\cdot u\rangle$	$-N\,F^{-6}\,c_d^3\,c_m + 4\,N\,F^{-6}\,c_d^4\,\lambda_3^{\rm SS} + N\,F^{-6}\,c_d^4 - N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} + N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} - 1/4\,N\,F^{-4}\,c_d\,c_m + 2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} + 2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 4\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} - 2\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} - 1/2\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - N\,F^{-2}\,(\lambda_1^{\rm SS})^2\,\lambda_3^{\rm SS} - N\,F^{-2}\,(\lambda_1^{\rm SS})^2 + N\,F^{-2}\,(\lambda_2^{\rm SS})^2 - 1/16\,N\,F^{-2}\,\lambda_3^{\rm SS}$
80	$\langle SS\ \{\chi_+,u\cdot u\}\rangle$	$-1/2\,N\,F^{-6}\,c_d^3\,c_m + 2\,N\,F^{-6}\,c_d^4\,\lambda_3^{\rm SS} + N\,F^{-6}\,c_d^4 - 1/2\,N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} + 1/2\,N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} - 1/8\,N\,F^{-4}\,c_d\,c_m + N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} + 1/2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 2\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} - N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 3/4\,N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 1/4\,N\,F^{-4}\,c_d^2 - 1/2\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - 1/8\,N\,F^{-2}\,\lambda_1^{\rm SS} - 1/2\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2 + 1/8\,N\,F^{-2}\,\lambda_3^{\rm SS}$

Table 3: (*continued*)

81	$\langle u_\nu S u^\nu \, \{S, \chi_+\}\rangle$	$1/2\, N F^{-4}\, c_d\, c_m\, \lambda_1^{\mathrm{SS}} - N F^{-4}\, c_d\, c_m\, \lambda_2^{\mathrm{SS}} + 1/8\, N F^{-4}\, c_d\, c_m - N F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}} + 4\, N F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 2\, N F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} - N F^{-4}\, c_d^2\, \lambda_3^{\mathrm{SS}} - 1/4\, N F^{-4}\, c_d^2 - 2\, N F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 1/2\, N F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}} + 1/2\, N F^{-2}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 1/8\, N F^{-2}\, \lambda_2^{\mathrm{SS}}$
82	$\langle u_\nu S S u^\nu \, \chi_+\rangle$	$-4\, N F^{-6}\, c_d^4\, \lambda_3^{\mathrm{SS}} + 2\, N F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 8\, N F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 1/8\, N F^{-2}\, \lambda_1^{\mathrm{SS}} + 1/4\, N F^{-2}\, (\lambda_1^{\mathrm{SS}})^2 - 4\, N F^{-2}\, (\lambda_2^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} - 3/64\, N F^{-2}$
83	$\langle u_\mu \rangle\, \langle u^\mu \, \{SS, \chi_+\}\rangle$	$3/64 F^{-2} - F^{-6}\, c_d^3\, c_m + 2\, F^{-6}\, c_d^4 + 1/2\, F^{-4}\, c_d\, c_m\, \lambda_1^{\mathrm{SS}} + 1/2\, F^{-4}\, c_d\, c_m\, \lambda_2^{\mathrm{SS}} + 1/4\, F^{-4}\, c_d\, c_m - 2\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}} + 2\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} - 1/2\, F^{-4}\, c_d^2 - 2\, F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 1/4\, F^{-2}\, \lambda_1^{\mathrm{SS}} + 1/4\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2 - 1/2\, F^{-2}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 1/4\, F^{-2}\, \lambda_3^{\mathrm{SS}}$
84	$\langle u_\mu \rangle\, \langle u^\mu \, S\, \chi_+\, S\rangle$	$-2\, F^{-6}\, c_d^3\, c_m + 8\, F^{-6}\, c_d^4\, \lambda_3^{\mathrm{SS}} + 2\, F^{-6}\, c_d^4 + F^{-4}\, c_d\, c_m\, \lambda_1^{\mathrm{SS}} + F^{-4}\, c_d\, c_m\, \lambda_2^{\mathrm{SS}} + 1/2\, F^{-4}\, c_d\, c_m - 4\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 2\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}} - 4\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 2\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} - 3\, F^{-4}\, c_d^2\, \lambda_3^{\mathrm{SS}} + F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}} + 2\, F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} - 1/4\, F^{-2}\, \lambda_2^{\mathrm{SS}} + 1/8\, F^{-2}\, \lambda_3^{\mathrm{SS}}$
85	$\langle u_\mu\, S\rangle\, \langle u^\mu \, \{S, \chi_+\}\rangle$	$-3/32 F^{-2} - 4\, F^{-6}\, c_d^3\, c_m + 6\, F^{-6}\, c_d^4 + 2\, F^{-4}\, c_d\, c_m\, \lambda_1^{\mathrm{SS}} + 2\, F^{-4}\, c_d\, c_m\, \lambda_2^{\mathrm{SS}} + F^{-4}\, c_d\, c_m + 4\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 4\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}} + 4\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 6\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} - 2\, F^{-4}\, c_d^2\, \lambda_3^{\mathrm{SS}} - F^{-4}\, c_d^2 - 4\, F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}} - 2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} + 1/2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2 + 3\, F^{-2}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 3/4\, F^{-2}\, \lambda_2^{\mathrm{SS}} - 4\, F^{-2}\, (\lambda_2^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} + F^{-2}\, (\lambda_2^{\mathrm{SS}})^2 - 1/8\, F^{-2}\, \lambda_3^{\mathrm{SS}}$
86	$\langle u_\mu\, SS\rangle\, \langle u^\mu \, \chi_+\rangle$	$3/32 F^{-2} - 4\, F^{-6}\, c_d^3\, c_m - 8\, F^{-6}\, c_d^4\, \lambda_3^{\mathrm{SS}} + 6\, F^{-6}\, c_d^4 + 2\, F^{-4}\, c_d\, c_m\, \lambda_1^{\mathrm{SS}} + 2\, F^{-4}\, c_d\, c_m\, \lambda_2^{\mathrm{SS}} + F^{-4}\, c_d\, c_m + 12\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 6\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}} + 8\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 4\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} + F^{-4}\, c_d^2\, \lambda_3^{\mathrm{SS}} - F^{-4}\, c_d^2 - 4\, F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}} + 1/2\, F^{-2}\, \lambda_1^{\mathrm{SS}} - 2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} + 1/2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2 - F^{-2}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 1/4\, F^{-2}\, \lambda_2^{\mathrm{SS}} - 3/8\, F^{-2}\, \lambda_3^{\mathrm{SS}}$
87	$\langle S\rangle\, \langle S\, \{\chi_+, u \cdot u\}\rangle$	$-2\, F^{-6}\, c_d^3\, c_m + 4\, F^{-6}\, c_d^4\, \lambda_3^{\mathrm{SS}} + 3\, F^{-6}\, c_d^4 - 1/2\, F^{-4}\, c_d\, c_m\, \lambda_1^{\mathrm{SS}} + F^{-4}\, c_d\, c_m\, \lambda_2^{\mathrm{SS}} - 5/8\, F^{-4}\, c_d\, c_m + F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}} - 2\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 3\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} + F^{-4}\, c_d^2\, \lambda_3^{\mathrm{SS}} + 3/4\, F^{-4}\, c_d^2 - 1/2\, F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}} - F^{-2}\, (\lambda_1^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} - F^{-2}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 3/8\, F^{-2}\, \lambda_2^{\mathrm{SS}} + 1/16\, F^{-2}\, \lambda_3^{\mathrm{SS}}$
88	$\langle S\, \chi_+\rangle\, \langle S\, u \cdot u\rangle$	$3/32 F^{-2} - 4\, F^{-6}\, c_d^3\, c_m + 6\, F^{-6}\, c_d^4 - F^{-4}\, c_d\, c_m + 12\, F^{-4}\, c_d^2\, \lambda_1^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} + 8\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 2\, F^{-4}\, c_d^2\, \lambda_2^{\mathrm{SS}} + F^{-4}\, c_d^2\, \lambda_3^{\mathrm{SS}} + F^{-4}\, c_d^2 - 4\, F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - F^{-2}\, \lambda_1^{\mathrm{SS}}\, \lambda_2^{\mathrm{SS}} - 2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2\, \lambda_3^{\mathrm{SS}} + 1/2\, F^{-2}\, (\lambda_1^{\mathrm{SS}})^2 - 3\, F^{-2}\, \lambda_2^{\mathrm{SS}}\, \lambda_3^{\mathrm{SS}} - 3/4\, F^{-2}\, \lambda_2^{\mathrm{SS}} - 2\, F^{-2}\, (\lambda_2^{\mathrm{SS}})^2 + 1/8\, F^{-2}\, \lambda_3^{\mathrm{SS}}$

Table 3: (*continued*)

89	$\langle SS\rangle\,\langle \chi_+\,u\cdot u\rangle$	$-3/64F^{-2}\,-\,2\,F^{-6}\,c_d^3\,c_m\,-\,4\,F^{-6}\,c_d^4\,\lambda_3^{\rm SS}\,+\,3\,F^{-6}\,c_d^4\,+\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,-\,3/4\,F^{-4}\,c_d\,c_m\,+\,6\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,+\,3/2\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,+\,F^{-4}\,c_d^2\,-\,1/2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,3/8\,F^{-2}\,\lambda_1^{\rm SS}\,-\,F^{-2}\,(\lambda_1^{\rm SS})^2\,\lambda_3^{\rm SS}\,+\,1/4\,F^{-2}\,(\lambda_1^{\rm SS})^2+3/16\,F^{-2}\,\lambda_3^{\rm SS}$
90	$\langle SS\,u\cdot u\rangle\,\langle \chi_+\rangle$	$-3/64F^{-2}\,-\,2\,F^{-6}\,c_d^3\,c_m\,-\,4\,F^{-6}\,c_d^4\,\lambda_3^{\rm SS}\,+\,3\,F^{-6}\,c_d^4\,-\,2\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,+\,2\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS}\,-\,1/2\,F^{-4}\,c_d\,c_m\,+\,2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,+\,3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,+\,8\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}\,-\,4\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,+\,1/2\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,+\,1/2\,F^{-4}\,c_d^2\,+\,1/2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,1/8\,F^{-2}\,\lambda_1^{\rm SS}\,-\,F^{-2}\,(\lambda_1^{\rm SS})^2\,\lambda_3^{\rm SS}\,-\,7/4\,F^{-2}\,(\lambda_1^{\rm SS})^2\,-\,4\,F^{-2}\,(\lambda_2^{\rm SS})^2\,\lambda_3^{\rm SS}\,+\,F^{-2}\,(\lambda_2^{\rm SS})^2\,+\,3/16\,F^{-2}\,\lambda_3^{\rm SS}$
91	$\langle SS\,\chi_+\rangle\,\langle u\cdot u\rangle$	$-3/64F^{-2}\,-\,2\,F^{-6}\,c_d^3\,c_m\,+\,4\,F^{-6}\,c_d^4\,\lambda_3^{\rm SS}\,+\,3\,F^{-6}\,c_d^4\,+\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,-\,3/4\,F^{-4}\,c_d\,c_m\,-\,2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,3\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,+\,9/2\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,+\,F^{-4}\,c_d^2\,-\,5/2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,3/8\,F^{-2}\,\lambda_1^{\rm SS}\,-\,F^{-2}\,(\lambda_1^{\rm SS})^2\,\lambda_3^{\rm SS}\,+\,1/4\,F^{-2}\,(\lambda_1^{\rm SS})^2+3/16\,F^{-2}\,\lambda_3^{\rm SS}$
92	$\langle u_\nu S u^\nu\,\chi_+\rangle\,\langle S\rangle$	$3/32F^{-2}\,-\,8\,F^{-6}\,c_d^4\,\lambda_3^{\rm SS}\,+\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,-\,2\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS}\,+\,1/4\,F^{-4}\,c_d\,c_m\,+\,4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,+\,12\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}\,+\,4\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,+\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,-\,1/2\,F^{-4}\,c_d^2\,-\,4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}\,+\,1/2\,F^{-2}\,(\lambda_1^{\rm SS})^2-F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}-2\,F^{-2}\,(\lambda_2^{\rm SS})^2$
93	$\langle u_\nu S u^\nu\,S\rangle\,\langle \chi_+\rangle$	$F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,-\,2\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS}\,+\,1/4\,F^{-4}\,c_d\,c_m\,+\,4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,+\,4\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,-\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,-\,1/2\,F^{-4}\,c_d^2\,-\,4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}\,-\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}+F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}+1/4\,F^{-2}\,\lambda_2^{\rm SS}$
94	$\langle \chi_+\,\nabla^\mu S\,\nabla_\mu S\rangle$	$1/2\,N\,F^{-4}\,c_d^2-N\,F^{-2}\,\lambda_1^{\rm SS}$
95	$\langle \chi_+\rangle\,\langle \nabla^\mu S\,\nabla_\mu S\rangle$	$1/2\,F^{-4}\,c_d^2-F^{-2}\lambda_1^{\rm SS}$
96	$\langle \chi_+\,\nabla^\mu S\rangle\,\langle \nabla_\mu S\rangle$	$F^{-4}\,c_d^2-2\,F^{-2}\,\lambda_2^{\rm SS}$
97	$i\,\langle u^\mu\,\nabla_\mu S\,\chi_-\,S\,\rangle\,+\,$h.c.	$-15\,N\,F^{-6}\,c_d^3\,c_m\,+\,9\,N\,F^{-6}\,c_d^4\,+\,4\,N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,+\,14\,N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS}+3\,N\,F^{-4}\,c_d\,c_m-3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,-\,10\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,+\,N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,-\,2\,N\,F^{-4}\,c_d^2\,+\,2\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}-2\,N\,F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS}+N\,F^{-2}\,\lambda_2^{\rm SS}$
98	$i\,\langle u^\mu\,\nabla_\mu S\,S\,\chi_-\,\rangle\,+\,$h.c.	$-13\,N\,F^{-6}\,c_d^3\,c_m\,+\,8\,N\,F^{-6}\,c_d^4\,+\,6\,N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,+\,10\,N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS}\,+\,5/2\,N\,F^{-4}\,c_d\,c_m\,-\,9/2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,-\,7\,N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,+\,2\,N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,-\,15/8\,N\,F^{-4}\,c_d^2\,-\,N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,+\,1/2\,N\,F^{-2}\,\lambda_1^{\rm SS}\,+\,1/2\,N\,F^{-2}\,(\lambda_1^{\rm SS})^2\,+\,2\,N\,F^{-2}\,(\lambda_2^{\rm SS})^2\,-\,3/4\,N\,F^{-2}\,\lambda_3^{\rm SS}\,+\,3/32\,N\,F^{-2}$
99	$i\,\langle u^\mu\,\chi_-\,\nabla_\mu S\,S\,\rangle\,+\,$h.c.	$2\,N\,F^{-6}\,c_d^3\,c_m\,-\,N\,F^{-6}\,c_d^4\,+\,3\,N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}\,-\,2\,N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS}\,-\,5/4\,N\,F^{-4}\,c_d\,c_m\,-\,3/2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS}+N\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}+5/8\,N\,F^{-4}\,c_d^2$
100	$i\,\langle u^\mu\rangle\,\langle \nabla_\mu S\,\{S,\chi_-\}\rangle$	$6\,F^{-6}\,c_d^3\,c_m-4\,F^{-6}\,c_d^4+F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS}-13/4\,F^{-4}\,c_d\,c_m-F^{-4}\,c_d^2\,\lambda_1^{\rm SS}\,+\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS}\,-\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS}\,+\,5/2\,F^{-4}\,c_d^2\,+\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS}\,-\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}\,-\,1/4\,F^{-2}\,\lambda_1^{\rm SS}\,+\,1/2\,F^{-2}\,(\lambda_1^{\rm SS})^2-3/4\,F^{-2}\,\lambda_2^{\rm SS}+3/4\,F^{-2}\,\lambda_3^{\rm SS}-3/32\,F^{-2}$

Table 3: (*continued*)

101	$i\,\langle u^\mu\,\{S,\chi_-\}\rangle\,\langle\nabla_\mu S\rangle$	$6\,F^{-6}\,c_d^3\,c_m - 4\,F^{-6}\,c_d^4 + 3\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} - 4\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} - 11/4\,F^{-4}\,c_d\,c_m - 2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} + 3\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 9/4\,F^{-4}\,c_d^2 + F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - 1/4\,F^{-2}\,\lambda_1^{\rm SS} + 1/2\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 3/4\,F^{-2}\,\lambda_2^{\rm SS} + 3/4\,F^{-2}\,\lambda_3^{\rm SS} - 3/32F^{-2}$
102	$i\,\langle\chi_-\,\nabla_\mu S\rangle\,\langle u^\mu\,S\rangle$	$8\,F^{-6}\,c_d^3\,c_m - 4\,F^{-6}\,c_d^4 + 2\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} - 8\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} - 5/2\,F^{-4}\,c_d\,c_m + 4\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} - 2\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + F^{-4}\,c_d^2 - 2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - F^{-2}\,\lambda_1^{\rm SS} + F^{-2}\,(\lambda_1^{\rm SS})^2 + 3/2\,F^{-2}\,\lambda_3^{\rm SS} + 3/16F^{-2}$
103	$i\,\langle S\rangle\,\langle\chi_-\,\{u^\mu,\nabla_\mu S\}\rangle$	$-4\,F^{-6}\,c_d^3\,c_m + 2\,F^{-6}\,c_d^4 + 5\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} + 2\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} + 1/4\,F^{-4}\,c_d\,c_m - 4\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 3\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 1/4\,F^{-4}\,c_d^2 + F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} + 1/4\,F^{-2}\,\lambda_1^{\rm SS} + 1/2\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 2\,F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} - 1/4\,F^{-2}\,\lambda_2^{\rm SS} - 3/4\,F^{-2}\,\lambda_3^{\rm SS} - 3/32F^{-2}$
104	$i\,\langle S\,\nabla_\mu S\rangle\,\langle u^\mu\,\chi_-\rangle$	$8\,F^{-6}\,c_d^3\,c_m - 4\,F^{-6}\,c_d^4 - 2\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} - 7/2\,F^{-4}\,c_d\,c_m + 2\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 2\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 3/2\,F^{-4}\,c_d^2 - 2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - F^{-2}\,\lambda_1^{\rm SS} + F^{-2}\,(\lambda_1^{\rm SS})^2 + 3/2\,F^{-2}\,\lambda_3^{\rm SS} + 3/16F^{-2}$
105	$i\,\langle S\,\{u^\mu,\nabla_\mu S\}\rangle\,\langle\chi_-\rangle$	$-20\,F^{-6}\,c_d^3\,c_m + 10\,F^{-6}\,c_d^4 + 11\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} + 18\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} + 11/4\,F^{-4}\,c_d\,c_m - 7\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 9\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 3\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} - F^{-4}\,c_d^2 + F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} + 1/4\,F^{-2}\,\lambda_1^{\rm SS} + 1/2\,F^{-2}\,(\lambda_1^{\rm SS})^2 - 2\,F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} - 1/4\,F^{-2}\,\lambda_2^{\rm SS} - 3/4\,F^{-2}\,\lambda_3^{\rm SS} - 3/32F^{-2}$
106	$i\,\langle S\,\chi_-\rangle\,\langle u^\mu\,\nabla_\mu S\rangle$	$-20\,F^{-6}\,c_d^3\,c_m + 16\,F^{-6}\,c_d^4 + 14\,F^{-4}\,c_d\,c_m\,\lambda_1^{\rm SS} + 12\,F^{-4}\,c_d\,c_m\,\lambda_2^{\rm SS} + 7/2\,F^{-4}\,c_d\,c_m - 12\,F^{-4}\,c_d^2\,\lambda_1^{\rm SS} - 16\,F^{-4}\,c_d^2\,\lambda_2^{\rm SS} + 6\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} - 4\,F^{-4}\,c_d^2 + 4\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_2^{\rm SS} - 2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} + F^{-2}\,\lambda_1^{\rm SS} + F^{-2}\,(\lambda_1^{\rm SS})^2 - 4\,F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} + 2\,F^{-2}\,\lambda_2^{\rm SS} + 4\,F^{-2}\,(\lambda_2^{\rm SS})^2 - 3/2\,F^{-2}\,\lambda_3^{\rm SS} + 3/16F^{-2}$
107	$\langle SS\,\chi_+\,\chi_+\rangle$	$-N\,F^{-4}\,c_d\,c_m\,\lambda_3^{\rm SS} - 1/2\,N\,F^{-4}\,c_d\,c_m + 2\,N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 2\,N\,F^{-4}\,c_d^2\,(\lambda_3^{\rm SS})^2 + 1/2\,N\,F^{-4}\,c_d^2 + 1/8\,N\,F^{-4}\,c_m^2 - N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - 1/4\,N\,F^{-2}\,\lambda_1^{\rm SS} + 1/4\,N\,F^{-2}\,\lambda_3^{\rm SS}$
108	$\langle S\,\chi_+\,S\,\chi_+\rangle$	$-N\,F^{-4}\,c_d\,c_m\,\lambda_3^{\rm SS} - 1/2\,N\,F^{-4}\,c_d\,c_m + 2\,N\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 2\,N\,F^{-4}\,c_d^2\,(\lambda_3^{\rm SS})^2 + 1/4\,N\,F^{-4}\,c_d^2 + 1/8\,N\,F^{-4}\,c_m^2 - N\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS}$
109	$\langle S\rangle\,\langle S\,\chi_+\,\chi_+\rangle$	$-2\,F^{-4}\,c_d\,c_m\,\lambda_3^{\rm SS} - 2\,F^{-4}\,c_d\,c_m + 4\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 3/2\,F^{-4}\,c_d^2 + 1/2\,F^{-4}\,c_m^2 - 2\,F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} - 1/2\,F^{-2}\,\lambda_2^{\rm SS}$
110	$\langle S\,\chi_+\rangle^2$	$-2\,F^{-4}\,c_d\,c_m\,\lambda_3^{\rm SS} - 2\,F^{-4}\,c_d\,c_m + 4\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 4\,F^{-4}\,c_d^2\,(\lambda_3^{\rm SS})^2 + 3/2\,F^{-4}\,c_d^2 + 1/2\,F^{-4}\,c_m^2 - 2\,F^{-2}\,\lambda_2^{\rm SS}\,\lambda_3^{\rm SS} - 1/2\,F^{-2}\,\lambda_2^{\rm SS}$
111	$\langle SS\rangle\,\langle\chi_+\,\chi_+\rangle$	$-F^{-4}\,c_d\,c_m + 3/4\,F^{-4}\,c_d^2 + 1/4\,F^{-4}\,c_m^2 - 1/4\,F^{-2}\,\lambda_1^{\rm SS} + 1/4\,F^{-2}\,\lambda_3^{\rm SS}$
112	$\langle SS\,\chi_+\rangle\,\langle\chi_+\rangle$	$-2\,F^{-4}\,c_d\,c_m\,\lambda_3^{\rm SS} - 2\,F^{-4}\,c_d\,c_m + 4\,F^{-4}\,c_d^2\,\lambda_3^{\rm SS} + 3/2\,F^{-4}\,c_d^2 + 1/2\,F^{-4}\,c_m^2 - 2\,F^{-2}\,\lambda_1^{\rm SS}\,\lambda_3^{\rm SS} - 1/2\,F^{-2}\,\lambda_1^{\rm SS} + 1/2\,F^{-2}\,\lambda_3^{\rm SS}$

Table 3: (*continued*)

113	$\langle S\,\chi_-\,S\,\chi_-\rangle$	$-14\,N\,F^{-6}\,c_d^2\,c_m^2 + 22\,N\,F^{-6}\,c_d^3\,c_m - 8\,N\,F^{-6}\,c_d^4 - 6\,N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\mathrm{SS}} - 12\,N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\mathrm{SS}} + 6\,N\,F^{-4}\,c_d\,c_m\,\lambda_3^{\mathrm{SS}} - 3/2\,N\,F^{-4}\,c_d\,c_m + 3\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 6\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 4\,N\,F^{-4}\,c_d^2\,\lambda_3^{\mathrm{SS}} + 3/4\,N\,F^{-4}\,c_d^2 + 2\,N\,F^{-4}\,c_m^2\,\lambda_1^{\mathrm{SS}} + 4\,N\,F^{-4}\,c_m^2\,\lambda_2^{\mathrm{SS}} + 1/2\,N\,F^{-4}\,c_m^2 - N\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 2\,N\,F^{-2}\,\lambda_2^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} - 1/4\,N\,F^{-2}\,\lambda_2^{\mathrm{SS}}$
114	$\langle SS\,\chi_-\,\chi_-\rangle$	$-18\,N\,F^{-6}\,c_d^2\,c_m^2 + 26\,N\,F^{-6}\,c_d^3\,c_m - 9\,N\,F^{-6}\,c_d^4 - 10\,N\,F^{-4}\,c_d\,c_m\,\lambda_1^{\mathrm{SS}} - 12\,N\,F^{-4}\,c_d\,c_m\,\lambda_2^{\mathrm{SS}} + 6\,N\,F^{-4}\,c_d\,c_m\,\lambda_3^{\mathrm{SS}} - N\,F^{-4}\,c_d\,c_m + 9/2\,N\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 6\,N\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 4\,N\,F^{-4}\,c_d^2\,\lambda_3^{\mathrm{SS}} + 1/2\,N\,F^{-4}\,c_d^2 + 4\,N\,F^{-4}\,c_m^2\,\lambda_1^{\mathrm{SS}} + 4\,N\,F^{-4}\,c_m^2\,\lambda_2^{\mathrm{SS}} + 1/2\,N\,F^{-4}\,c_m^2 + N\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} - 1/8\,N\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/4\,N\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - N\,F^{-2}\,(\lambda_2^{\mathrm{SS}})^2 + 1/4\,N\,F^{-2}\,\lambda_3^{\mathrm{SS}} - N\,F^{-2}\,(\lambda_3^{\mathrm{SS}})^2 - 1/64\,N\,F^{-2}$
115	$\langle S\rangle\,\langle S\,\chi_-\,\chi_-\rangle$	$1/32F^{-2} - 16\,F^{-6}\,c_d^2\,c_m^2 + 20\,F^{-6}\,c_d^3\,c_m - 6\,F^{-6}\,c_d^4 - 12\,F^{-4}\,c_d\,c_m\,\lambda_1^{\mathrm{SS}} - 8\,F^{-4}\,c_d\,c_m\,\lambda_2^{\mathrm{SS}} + 12\,F^{-4}\,c_d\,c_m\,\lambda_3^{\mathrm{SS}} + 2\,F^{-4}\,c_d\,c_m + 6\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 3\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 8\,F^{-4}\,c_d^2\,\lambda_3^{\mathrm{SS}} - F^{-4}\,c_d^2 + 4\,F^{-4}\,c_m^2\,\lambda_1^{\mathrm{SS}} + 4\,F^{-4}\,c_m^2\,\lambda_2^{\mathrm{SS}} - F^{-4}\,c_m^2 - F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} - 1/2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 2\,F^{-2}\,\lambda_2^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} + 1/4\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 2\,F^{-2}\,(\lambda_3^{\mathrm{SS}})^2$
116	$\langle SS\rangle\,\langle\chi_-\,\chi_-\rangle$	$-1/64F^{-2} - 4\,F^{-6}\,c_d^2\,c_m^2 + 4\,F^{-6}\,c_d^3\,c_m - F^{-6}\,c_d^4 - 4\,F^{-4}\,c_d\,c_m\,\lambda_1^{\mathrm{SS}} + 4\,F^{-4}\,c_d\,c_m\,\lambda_3^{\mathrm{SS}} + 1/2\,F^{-4}\,c_d\,c_m + 3/2\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} - 2\,F^{-4}\,c_d^2\,\lambda_3^{\mathrm{SS}} - 1/4\,F^{-4}\,c_d^2 + 2\,F^{-4}\,c_m^2\,\lambda_1^{\mathrm{SS}} + F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} + 1/8\,F^{-2}\,\lambda_1^{\mathrm{SS}} - 1/4\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 - 1/4\,F^{-2}\,\lambda_3^{\mathrm{SS}} - F^{-2}\,(\lambda_3^{\mathrm{SS}})^2$
117	$\langle S\,\chi_-\rangle^2$	$-1/32F^{-2} - 20\,F^{-6}\,c_d^2\,c_m^2 + 28\,F^{-6}\,c_d^3\,c_m - 10\,F^{-6}\,c_d^4 - 12\,F^{-4}\,c_d\,c_m\,\lambda_1^{\mathrm{SS}} - 12\,F^{-4}\,c_d\,c_m\,\lambda_2^{\mathrm{SS}} + 12\,F^{-4}\,c_d\,c_m\,\lambda_3^{\mathrm{SS}} + F^{-4}\,c_d\,c_m + 6\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 7\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 8\,F^{-4}\,c_d^2\,\lambda_3^{\mathrm{SS}} + 4\,F^{-4}\,c_m^2\,\lambda_1^{\mathrm{SS}} + 4\,F^{-4}\,c_m^2\,\lambda_2^{\mathrm{SS}} - F^{-4}\,c_m^2 - F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} - 1/2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 2\,F^{-2}\,\lambda_2^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} - 1/4\,F^{-2}\,\lambda_2^{\mathrm{SS}} - F^{-2}\,(\lambda_2^{\mathrm{SS}})^2 - 2\,F^{-2}\,(\lambda_3^{\mathrm{SS}})^2$
118	$\langle SS\,\chi_-\rangle\,\langle\chi_-\rangle$	$1/32F^{-2} - 32\,F^{-6}\,c_d^2\,c_m^2 + 44\,F^{-6}\,c_d^3\,c_m - 14\,F^{-6}\,c_d^4 - 20\,F^{-4}\,c_d\,c_m\,\lambda_1^{\mathrm{SS}} - 20\,F^{-4}\,c_d\,c_m\,\lambda_2^{\mathrm{SS}} + 16\,F^{-4}\,c_d\,c_m\,\lambda_3^{\mathrm{SS}} - F^{-4}\,c_d\,c_m + 9\,F^{-4}\,c_d^2\,\lambda_1^{\mathrm{SS}} + 8\,F^{-4}\,c_d^2\,\lambda_2^{\mathrm{SS}} - 10\,F^{-4}\,c_d^2\,\lambda_3^{\mathrm{SS}} + 8\,F^{-4}\,c_m^2\,\lambda_1^{\mathrm{SS}} + 8\,F^{-4}\,c_m^2\,\lambda_2^{\mathrm{SS}} + F^{-4}\,c_m^2 - F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_2^{\mathrm{SS}} + 2\,F^{-2}\,\lambda_1^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} - 1/2\,F^{-2}\,(\lambda_1^{\mathrm{SS}})^2 + 2\,F^{-2}\,\lambda_2^{\mathrm{SS}}\,\lambda_3^{\mathrm{SS}} + 1/4\,F^{-2}\,\lambda_2^{\mathrm{SS}} - 2\,F^{-2}\,(\lambda_3^{\mathrm{SS}})^2$

Table 3: Operators with two scalar resonances and their β-function coefficients when only scalar resonances are included.

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